Rapid flow separation for transient inflow conditions versus accelerating bodies: An investigation into their equivalency

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A R T I C L E   I N F O

Article history:
Received 13 September 2012
Accepted 2 April 2013
Available online 6 June 2013

Keywords:
Flow separation
Added mass
Vortical flows

A B S T R A C T

The possibility of simulating gusts or flow disturbances by accelerating the body rather than the oncoming fluid has been investigated. In this study the effect of a two-dimensional compound gust is considered, defined as the variation of both axial and lateral flow disturbances. The force histories of gust-induced loads on a flat-plate model were compared to those on an equivalent model accelerated within a uniform flow. It was shown that the contribution to force history from fluid acceleration could not be neglected, even at a relatively low reduced frequency. However, differences between gust loads and accelerating-model loads were not proportional to model acceleration. Rather, when differences in the fluid velocity fields were examined, the force histories were differentiated primarily by the distribution of vorticity in the wake, caused by changes in the effective incidence and effective velocity at the trailing-edge of the model.

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1. Background

Natural fliers and swimmers, autonomous aerial and underwater vehicles, as well as propellers, rotors and other turbomachinery, all operate in highly unsteady flows leading to rapid flow separation and therefore large dynamic loadings. As described in Leishman (2006), classical unsteady theory assumes fully attached (potential) flow, and is fundamentally unable to describe aerodynamic or hydrodynamic loadings in these separated flow regimes. Gust response is important at all Reynolds numbers, such as for very large wind turbines as studied by Wächter et al. (2011), however, the lower moments of inertia associated with smaller vehicles such as Micro Aerial Vehicles (MAVs) and Autonomous Underwater Vehicles (AUVs) make them just that much more sensitive to disturbances. In addition to vehicle stability, flapping-wing performance is susceptible to gusts as well, as discussed by Jones and Yamaleev (2012). Research on active gust suppression in the context of MAV stability is ongoing, but current gust models are limited, see e.g. Kerstens et al. (2010) or Tadmor et al. (2008). Recently, Knebel et al. (2011), Shields and Mohseni (2010a, 2010b) and others have developed active grids to generate specific turbulent fields in wind tunnels. Such active grids can use dozens of individual stepper motors to operate, and as such a large investment of time is required for their implementation and verification, see Roadman and Mohseni (2009). Large arrays of computer fans have also been used to a similar effect, such as by Johnson and Jacob (2009). Producing well-defined gusts such as with loud speakers, as described by Olsman et al. (2010), can also be technically challenging and are only suitable for studies on flutter with high-frequency and small-amplitude disturbances. Other methods of gust generation explored in the past include oscillating upstream wings as used by Ham et al. (1974) and Rival et al. (2011), rotor-stator inlet devices as demonstrated by Al-Asmi and Castro (1993), and downstream shutters, as used by Kerstens et al. (2010).
As described by Roadman and Mohseni (2009), the gusts produced in such facilities must be characterized statistically, where instantaneous gust forms are very difficult to reproduce. However, the investigation of instantaneous loadings requires the direct knowledge of the instantaneous flow field, particularly of eddies in the order of the vehicle scale. Instead, it is reasonable to imagine that for some classes of in-flow disturbances, variations in the fluid flow are equivalent to opposing motions of a model in a steady flow. A simple Galilean transformation of variations in the flow field upstream of a model can have many potential advantages: the number of active components required to move a model in a prescribed path can be much smaller than that required to produce an equivalent fluid motion; paths of a manipulated model can also be more repeatable than the flow field of a gusty environment, and if a complex gust was to be simulated directly, it would not be necessary to interpolate or extrapolate between individual gust modes as described by Kramer (2002). However, it is unclear what class of gusts may be suitably simulated with a technique involving equivalent model motions. The purpose of this study is to characterize the differences between the cases of a stationary model subjected to a simplified gust, and a model accelerated within a uniform flow at corresponding rates.

1.1. Simplified gust profile

In this section, simplified gust profiles will be defined based on the qualitative effects of a passing vortex. Consider a roller vortex of strength $\Gamma$ embedded in a free-stream flow of velocity $U_\infty$, passing a profile from height of $\Delta y_0$, as shown in Fig. 1. The induced velocity $u_\phi$ and the corresponding angle $\phi$ of such a roller on a model at varying distance $r(t)$ becomes

$$|\bar{u}_\phi(t)| = \frac{\Gamma}{2\pi r(t)},$$

and

$$\phi(t) = \arctan\left(\frac{\Delta x(t)}{\Delta y_0}\right),$$

respectively (see Fig. 1(a)). A qualitative plot of the corresponding $\alpha_{\text{eff}}$ and $\bar{u}_{\text{eff}}$ is shown in Fig. 2.

If the slow ramp up to $a_{\text{eff}}^{\text{max}} (t < t^+)$ and the slow ramp down from $a_{\text{eff}}^{\text{max}} (t > t^-)$, as in Fig. 2, are considered to be quasi-steady, then it is the period of rapid change in effective incidence from $t^+$ to $t^-$ that will produce unsteady loadings. We may then use $t^+$ and $t^-$ to define a characteristic time scale $T$ of the passing gust, based on twice the time span between the

![Fig. 1. A roller vortex embedded in a uniform flow passing near a stationary model produces an induced velocity $u_\phi$ at that model (a). The stationary model experiences an effective velocity $u_{\text{eff}}$ as the vector sum of $u_\phi$ and $U_\infty$ (b). Both the magnitude of velocity $|\bar{u}_{\text{eff}}|$ and the effective incidence $\alpha_{\text{eff}}$ vary with time as the roller convects downstream. (a) Roller passing near model. (b) Velocity triangle at model.](image-url)
maximum and minimum effective angles of attack. Given this time scale \( T \), a gust wavelength of the passing roller can be defined as

\[
\lambda = \frac{U_\infty}{f} = \frac{U_\infty}{\frac{1}{T}},
\]

where \( f = \frac{1}{T} \) defines the characteristic frequency of the gust. Combining (3) with the definition of reduced frequency \( k = \frac{\pi c}{U_\infty} \), we obtain an equivalent definition of reduced frequency based on gust wavelength

\[
k = \frac{\pi c}{\lambda},
\]

where \( c \) is the profile chord.

Fig. 2. Before \( t^+ \), both the effective incidence \( \alpha_{\text{eff}} \) and effective velocity \( \overline{u}_{\text{eff}} \) induced on a model by a passing roller vortex change slowly, and can be treated with quasi-steady theory. Highly unsteady loadings would be expected only between \( t^+ \) and \( t^- \) when rapid changes in \( \alpha_{\text{eff}} \) and \( \overline{u}_{\text{eff}} \) occur. \( t^+ \) and \( t^- \) can thus be used to define a characteristic time \( T/2 \).

If the effect of short wavelength gusts on an aerodynamic profile with chord \( c \) is analogous to isotropic turbulence, this scale should have no net bearing on the aerodynamic loads since they are small in scale (i.e. \( \lambda/c \ll 1 \)) and should average out over the model, see Systsma and Ukeiley (2010). In contrast, long-wavelength gusts (\( \lambda/c \gg 1 \)) simply act to change the mean wind speed and angle, and can be treated with quasi-steady theory, i.e. \( C_{l} \approx 2 \alpha_{\text{eff}} \). As a result, only wavelengths in the order of one chord (\( \lambda = O(1) \)) should be significant to unsteady aerodynamic loadings on a profile.

Thus, we will consider two cases, each of which closely replicates the rapid changes of effective incidence in the range of \( t^+ < t < t^- \). A sinusoidal plunging motion is considered as the simplest case that replicates the effective incidence of a passing roller (hereafter referred to as a lateral gust, or by the subscript L). In order to understand the effects of a varying effective velocity \( u_{\text{eff}} \), as seen in Fig. 2, a second case will be considered where the effective velocity is maintained at \( \overline{u}_{\text{eff}} = U_\infty \), while maintaining the effective incidence of the plunging case (hereafter referred to as a lateral–axial gust, or by the subscript LA). These two cases are sketched in Fig. 3, where \( \dot{s} \) is defined as a velocity in the chordwise direction.

The equations of motion are therefore

\[
\dot{h}(t)|_{L} = h_0 \cos (2\pi ft),
\]

\[
\dot{s}(t)|_{L} = 0, \quad \text{and}
\]

\[
\alpha_{\text{eff}}(t)|_{L} = \arctan(h_0 \cos (2\pi ft)/U_\infty)
\]

for the lateral gust case, where \( h_0 \) is the plunge amplitude, and

\[
\dot{h}(t)|_{LA} = \sin (\alpha_{\text{eff}})|U_\infty, \quad \text{and}
\]

\[
\dot{s}(t)|_{LA} = (1 - \cos (\alpha_{\text{eff}}))U_\infty, \quad \text{and}
\]

\[
\alpha_{\text{eff}}(t)|_{LA} = \arctan(h_0 \cos (2\pi ft)/U_\infty)\equiv\alpha_{\text{eff}}(t)|_{L}
\]

for the lateral–axial case, where the frequency \( f \) is a function of the reduced frequency \( k \). Recalling that only gusts with a wavelength in the order of a chord (\( \lambda/c = O(1) \)) should be considered, we can limit ourselves to the largest wavelengths \( \lambda \) in this range, which are likely to contain the largest amount of energy. The largest wavelengths represent the smallest reduced frequencies, and therefore reduced frequencies near \( k = \pi/10 \) are characteristic of gusts whose effects are significant to unsteady loadings. Given this reduced frequency, a classical analysis can now be performed as a first-order approximation of the differences between a gusting flow and a moving profile.
1.2. Classical analysis

As discussed in Leishman (2006), Theodorsen’s theory was developed to investigate harmonic oscillations of a thin profile in a fully attached flow. As a first approximation this classical (potential) technique can therefore be used to study the lateral gust case discussed above, assuming that flow separation plays a minor role—see Ol et al. (2009). In order to simplify the analysis, Theodorsen decomposed lift into a linear superposition of circulatory and non-circulatory (added mass) components. If \( \alpha(t) \) oscillates around zero, this takes the form

\[
C_l = 2\pi C(k) \left( \frac{c}{U_\infty} + \frac{\dot{h}}{2U_\infty} \right) + \frac{\pi c}{2} \left( \frac{\dot{\alpha}}{U_\infty} + \frac{\dot{h}}{U_\infty} + \frac{c \ddot{\alpha}}{4U_\infty^2} \right).
\]  

(7)

where \( C(k) \) is a complex-valued function of the reduced frequency \( k \) known as the Theodorsen function. Eq. (7) can be evaluated for pure plunging, similar to the lateral gust profile

\[
h(t) = h \cos(2\pi ft), \quad \text{and} \quad \alpha(t) = 0.
\]  

(8)

(9)

Fig. 4(a) represents a comparison of lift coefficients for a pure-plunging motion with reduced frequency of \( k = 0.25 \), determined both with Theodorsen theory and from quasi-steady assumptions \( (C_l = 2\pi \alpha_{eff}) \). In Fig. 4(b) the contributions of the circulatory and non-circulatory (added mass) terms are compared. It can be seen that the effect of the non-circulatory part is small in comparison with that of the circulatory part, in that the amplitude of added mass contributions are small compared to the circulatory contribution. In other words the added-mass term is the only difference between a gust and equivalent model motions in Theodorson theory. Therefore one can infer that for situations where the non-circulatory contribution is small (e.g. low reduced frequencies), a direct Galilean transformation appears to be a valid method for simulating gusts with wavelengths near \( \lambda = 10c \).
2. Methods

In the previous section, classical analysis showed that added mass is the sole difference between a gust and equivalent model motions for the case of attached flows. Therefore, consider the decomposition of lift into two constituent components, as done with Theodorsen theory in Eq. (7)

\[ C_l = C_{\text{circ}} + C_{\text{am}}, \]  

(10)

where \( C_{\text{circ}} \) is the circulatory component of lift, and \( C_{\text{am}} \) represents acceleration effects analogous to the potential concept of added mass. Recently it has been shown by Pitt Ford and Babinsky (2013) that the bound circulation \( \Gamma_b \) can be neglected for a flat plate with leading-edge separation, such that lift is generated primarily by the LEV instead. As this study will consider a flat plate model with strong leading-edge separation, we may further decompose \( C_{\text{circ}} \) as

\[ C_{\text{circ}} = C_{\text{LEV}} + C_{\text{wake}}, \]  

(11)

where \( C_{\text{LEV}} \) is the quasi-steady component of lift, and \( C_{\text{wake}} \) is due to the near-wake vorticity distribution. In order to simulate a gust by moving a model, two conditions must be met: first, acceleration effects must be small, as these effects would be zero in the case of a gust; second, the model acceleration must not have a significant effect on \( C_{\text{LEV}} \) or \( C_{\text{wake}} \).

In order to test these conditions, the gust forms described in Eqs. (5) and (6) were simulated numerically in the laminar regime. Laminar simulations were chosen as a first approximation since unsteady, sharp plates are Reynolds-number insensitive – see Ol et al. (2009) – as well as to avoid errors associated with modeling transition and turbulence. The simulations for the “moving-fluid case”, referred to hereon in as MF, are then compared to simulations for a “moving-model case” (MM), where equivalent leading-edge velocities were produced by manipulating the model within a uniform flow. Finally, experiments for the MM case were performed in a water tunnel to both validate the robustness of the simulations as well as to confirm the degree of Reynolds-number insensitivity.

2.1. Numerical setup

In the numerical simulations, performed at \( Re = 1600 \), a flat plate was exposed to laminar, periodic lateral and compound lateral–axial gusts. These gusts are described by Eqs. (5) and (6), which are varied at the inlet boundary. The structured, quadrilateral mesh used for the simulation included approximately 200 000 cells spanning \( 20c \times 20c \) as shown in Fig. 5. The distance between two successive nodes in proximity of the plate is 0.005c for both directions. Simultaneously, lateral and compound lateral–axial plate motions were simulated, where an identical \( 20c \times 20c \) mesh was appended with a 3c-thick deforming zone around the original domain. The \( 20c \times 20c \) region was displaced in accordance with the velocities described by Eqs. (5) and (6) with a constant inlet condition. The continuity and momentum equations were solved with a commercial software package (ANSYS CFX 13), where a variable-order upwind scheme was used for the advection term and a second-order backward Euler scheme was applied for the transient term. The convergence criteria were set to \( 10^{-5} \) for the dimensionless residuals of all quantities. The coupled scheme was adopted for the coupling of velocity and pressure. The mesh and domain independency of the results for both varying temporal and spatial resolutions were checked in order to ensure that the physics of the problem was properly captured. A more detailed description of the numerical setup is available in Mohebbian and Rival (2012).

2.2. Experimental procedure

All experiments were conducted in a free-surface water tunnel at the University of Calgary, as sketched in Fig. 6. The first water-tunnel test section (D, Fig. 6) has a diverging rectangular cross section with a mean width of 385 mm. Water depth was maintained at 432 mm. An aluminium flat-plate model of 50 mm chord and 3 mm thickness was used in the experimental study. The flat-plate model pierced the free-surface, which was considered a mirror plane, as Williamson (1989) showed that perturbations from free-surface interactions can take several shedding cycles to develop into oblique

Fig. 5. A flat-plate model in the computational domain, with a magnified view of the structured mesh immediately around the plate. In moving-model cases, a deforming region was appended to the outside of this domain.
shedding modes. Furthermore, the tip-gap between the model and the tunnel floor was maintained at less than 3 mm (0.06c), in order to mitigate the effects of a free tip; see Slaouti and Gerrard (1981). The model was therefore assumed to behave two-dimensionally. The model kinematics for velocity profiles were produced with a six degree-of-freedom hexapod manipulator, which was mounted over top of the water-tunnel facilities as shown in Fig. 7(a). Two separate measurement techniques were used simultaneously to capture the desired flow features and will be briefly described below. For all experimental data, individual runs were ensemble averaged, and errors were estimated as the maximum difference between any two individual measurements.

An ATI Gamma six-component force/torque sensor (1000 Hz sample rate, 16-bit sample depth) was flanged between the hexapod and the flat plate to capture the total forces acting on the model. The recorded force data were further post-processed by means of a Butterworth low-pass (25 Hz cutoff) and a moving average (50 ms span) filter similar to that used by Jones and Babinsky (2011). The unsteady flow-field behavior was captured with a commercial high-speed particle image velocimetry (PIV) system (LaVision/DaVis7.2), comprised of a Photonics Industries Nd:YLF (λ = 527 nm) single-cavity laser and a Photron APX-RS high-speed camera (12-bit monochrome image of 1024 × 1024 pixel resolution), as sketched in Fig. 7(b). The flow was seeded with 100 μm silver-coated hollow glass spheres. The seeding particles have a Stokes number of approximately 2.4 × 10^{-3}. The PIV system was operated in single-frame mode at a repetition rate of 250 frames per second and a pulse duration of 350 ns, to capture 393 snap shots per plunge cycle.

Flow fields were calculated with a multi-grid/multi-pass cross-correlation algorithm. Square interrogation areas with a final/initial length of 16/128 pixels and 75% overlap were chosen to retain the velocity gradients in the flow patterns. Outliers (< 5%) were eliminated using a neighborhood validation with a 3 × 3 moving average. Finally, each interrogation area was temporally filtered with a second-order Savitzky–Golay filter (125 ms span) to improve the salience of the coherent structures. Based on a chosen field of view of approximately 100 × 100 mm and full camera resolution, the setup resulted in approximately 100 datums per chord.

Fig. 6. Planview of the free-surface water tunnel used for the experiments. The tests were conducted in the middle of test section D at a chord-based Reynolds number of Re = 10 000.

Fig. 7. Description of the experimental apparatus: (a) The hexapod manipulator with high aspect-ratio flat plate model and 6-component force/torque balance positioned in the free-surface water tunnel; (b) a detailed sketch of the experimental setup showing the coordinate system, quasi-2D flat-plate model, and PIV field of view. (a) Hexapod manipulator. (b) PIV setup.
3. Results and discussion

In the following we test our previous hypotheses that: (1) acceleration effects are small relative to circulatory effects for lower reduced frequencies; and (2) if differences occur then acceleration effects are the dominant difference between the MF and MM cases. In addition, we first compare our laminar simulations to turbulent experiments in order to check the robustness of our simulations and also to verify that flow-field evolution is not Reynolds-number dependent. In such a way the conclusions drawn from the laminar simulations can then be generalized to higher Reynolds numbers.

3.1. Reynolds-number independence

As gusts can be simulated numerically in the laminar regime with relative ease, the motivation for future experimental MM gust simulations is the study of transitional or turbulent flows on Reynolds-number sensitive shapes. In this current section, however, a comparison of experimental cases (Re = 10 000) and numerical cases (Re = 1600) is performed for two
reasons: (1) to check the robustness of the simulations; (2) to verify that Reynolds-number independence is indeed satisfied for sharp two-dimensional plates. Vorticity contours captured for two-dimensional flat plates, plunging with identical reduced frequency $k$ and effective incidence $\alpha_{eff}$, are shown in Fig. 8 at four timesteps and for Reynolds numbers of $Re = 1600$ in the left column, and $Re = 10,000$ on the right. Although Reynolds numbers differ by nearly an order of magnitude, temporal and spacial evolution of the LEV and TEV are very similar at all timesteps, including detachment and convection of the vortices.

In order to quantify this similarity between computation and experiment, a circulation history was produced as a more robust measure of the flow field. Vorticity was integrated within a $1c/C_2$ interrogation window immediately above the model, resulting in the circulation histories in Fig. 9(a), where clockwise circulation is defined as positive. It was expected that circulation computed from PIV data would be lower than that computed from computational data, due to the filter-like nature of the PIV cross correlation. However, circulation histories follow closely, including a rapid increase in counterclockwise circulation near $t/T = 0.35$ corresponding to the formation of a trailing-edge vortex (TEV), and a simultaneous reduction in clockwise circulation corresponding to the primary leading-edge vortex (LEV) convecting out of the interrogation window. A comparison of the force histories for the two Reynolds numbers, shown in Fig. 9(b), also shows a very similar trend. The lack of Reynolds number dependence is also present in the lateral–axial gust case, shown in the force and circulation histories of Figs. 10 and 12, respectively. We therefore conclude that vortex evolution is independent of Reynolds number, at least within the range of $1000 \leq Re \leq 10,000$, which is consistent with the findings of OL et al. (2009) and Baik et al. (2012).

![Fig. 9. Circulation histories (left) and lift-force histories (right) for laminar (Re = 1600) and transitional (Re = 10,000) regimes are similar throughout the measured half-cycle. For experimental cases, error bars are plotted every 15th data point for circulation history, and every 150 data points for force history. (a) Circulation history. (b) Lift-force history.](image)

![Fig. 10. A comparison of moving-model and moving-fluid lift-force histories for a purely lateral gust (left) and for a compound axial–lateral gust (right). Near $t/T = 0$, lift force is higher for moving-model cases. However, the difference between force histories is not proportional to model acceleration for most of the range $0 \leq t/T \leq 0.5$. For experimental cases, error bars are plotted every 150 data points. (a) Lateral gust. (b) Axial–lateral gust.](image)
3.2. Acceleration effects

The acceleration of fluid near a moving model has many parallels with the inviscid concept of added mass discussed previously. As with added mass, fluid acceleration produces a contribution to force history in-phase with acceleration. In Fig. 10, we compare the force histories of MM and MF cases, for both lateral (Fig. 10(a)) and lateral–axial (Fig. 10(b)) gust profiles. Note that we have retroactively defined \( t/T = 0 \) to be the time when the gust front convects past the quarter-chord position for the moving fluid cases, as this minimizes differences between the flow fields of MF and MM cases, as seen in Fig. 11. Immediately after \( t/T = 0 \) when the motion begins, MM cases rapidly come to a lift coefficient of \( C_L = 0.5 \), which is significantly higher than the MF cases. The flow-field is essentially unchanged at that point in time, and consequently this force must be due to some acceleration effects. Furthermore, the lateral acceleration \( h \) is very similar between lateral and axial–lateral cases (within 1%), which results in their nearly identical lift at \( t/T = 0 \). However, the instantaneous difference in force history between MF and MM cases does not appear to be proportional to the instantaneous acceleration of the model in the MM cases. At \( t/T = 0.5 \), when accelerations are large, force histories converge. Conversely, large differences in lift are observed between MF and MM cases near \( t/T = 0.25 \), when accelerations are zero. Moreover, when these large differences in

![Fig. 11. Dimensionless vorticity \((\omega c/U_\infty)\) contours for dimensionless times 0.1, 0.2, 0.3, and 0.4 for a moving-model (left) and moving-fluid (right) cases of a lateral gust profile (CFD, Re = 1600). The differences in the instantaneous separated regions are very small when comparing the two cases.](image)
MF cases, as well as lateral and lateral model. In turn, force histories can be related the effective incidence differences observed in force history can therefore be related to the rate that wake vorticity is convected away from the model and the counter-clockwise vorticity in the wake would have a large impact on the magnitude of this downwash. The between MF and MM cases near subtraction of MM and MF vorticity fields (et al. (2012), who showed effective incidence circular effects

lift are observed at $t/T=0.25$, the MM case produces much more lift than the MF case for a lateral gust, but conversely less lift for the lateral–axial gust. We must therefore conclude that the assumption that acceleration effects are the only significant difference between MF and MM cases is incorrect. The collapse of experimental and computational MM cases for both lateral and lateral–axial gusts is a strong indication that the differences between MM and MF force histories would also be observed at $Re=10 000$. To uncover the observed discrepancies, a more rigorous analysis of the differences found in the flow-field is provided in the following section.

3.3. Circulatory effects

The flow-field for both MM and MF cases is very similar, as shown in Fig. 11 for the case of a pure lateral gust. Small differences near the trailing edge can be observed, but differences in the bulk flow are difficult to identify. Contours for the compound lateral–axial case were omitted as they show the identical trend. Similarly, the circulation histories for MM and MF cases, as well as lateral and lateral–axial gust profiles, are nearly indistinguishable as demonstrated in Fig. 12, even when extended to a Reynolds number of $Re=10 000$ for the experimental MM cases. This is consistent with the findings of Baik et al. (2012), who showed effective incidence $\alpha_{\text{eff}}$ and reduced frequency $k$ to be the primary factors in flow-field evolution.

So far, it has been shown that neither acceleration effects nor net circulation is responsible for the differences observed between MF and MM cases near $t/T=0.25$. As such, to investigate the small variations in the vorticity field, a simple subtraction of MM and MF vorticity fields $(\omega_{\text{MF}}-\omega_{\text{MM}})$ was performed. The resulting vorticity differences are shown in Fig. 13 for timesteps $t/T=0.2$ and $t/T=0.25$, while there are small differences in the shape and size of the LEV, a large difference is observed in the way trailing-edge vorticity is shed into the wake. These differences in trailing-edge vorticity are analogous to those observed by Prangemeier et al. (2010) for a superimposed quick-pitch motion. Similar variations are seen in the wake for both lateral and lateral–axial gust forms, where the counter-clockwise vorticity shed in the MF case (red streak in the wake of Fig. 13) is observed to have a lower angle of departure than in the MM case (blue streak in the wake of Fig. 13). Both lateral and lateral–axial cases show similar patterns of vorticity-field variation $\omega_{\text{MF}}-\omega_{\text{MM}}$ in their wakes. However, careful inspection reveals that this pattern is shifted upstream relative to the model by approximately 0.25c for the lateral–axial case. By elimination, we can conclude that the distribution of trailing-edge vorticity in the wake has a profound influence on the instantaneous force history.

Having determined that the distribution of wake vorticity has a significant impact on force history, we can attempt to explain the nature of that relationship. The vorticity shed into the wake is primarily counter-clockwise, and therefore produces a downwash on the model which reduces lift. It follows from the Biot–Savart law that the distance between the model and the counter-clockwise vorticity in the wake would have a large impact on the magnitude of this downwash. The differences observed in force history can therefore be related to the rate that wake vorticity is convected away from the model. In turn, force histories can be related the effective incidence $\alpha_{\text{eff}}$ and effective velocity $\vec{u}_{\text{eff}}$, evaluated at the trailing edge of the model.

In the MF cases, effective incidence $\alpha_{\text{eff}}$ and effective velocity $\vec{u}_{\text{eff}}$ do not immediately change at the trailing edge of the model, but rather only after the gust has had time to convect across the model instead. The rate that vorticity is convected from the trailing edge is affected by this convective delay, as illustrated in Fig. 14, where the convective of a fluid particle from the trailing edge of the model is compared between MF and MM cases. In Fig. 14, the distance $R$ between the model and a fluid particle convected from its trailing-edge differs between MF and MM cases due to differences in effective incidence $\alpha_{\text{eff}}$ and effective velocity $\vec{u}_{\text{eff}}$ at the trailing edge. For a lateral gust, the convected distance of the fluid particle is greater in the MM case, $R_{\text{MF}} < R_{\text{MM}}$, which coincides with the larger lift force for the MM case. Conversely, for the compound
lateral–axial gust, the convected distance of the fluid particle is less in the MM case, \( R_{MF} > R_{MM} \), which coincides with the smaller lift force in the MM case. This finding reveals an important insight, as it demonstrates that the time required for a gust to convect across a model cannot be neglected in gust simulations at these relatively low reduced frequencies.

4. Conclusions

In this study, the possibility of simulating gusts by accelerating the body rather than the oncoming fluid has been investigated. Two gust profiles were considered: a one-dimensional gust was defined as a variation in lateral flow velocities; and a two-dimensional compound gust was defined as the variation of both axial and lateral flow velocities. The variation in effective incidence \( \alpha_{eff} \) and reduced frequency \( k = 0.25 \) were held constant for all cases. Two major conclusions have been drawn. First, it has been shown that acceleration effects cannot be neglected despite relatively low reduced frequencies, as large differences in instantaneous force were observed between MM and MF cases. These discrepancies in instantaneous lift coincided with the high accelerations early in the cycle near \( t/T = 0.25 \). Furthermore, the assumption that such acceleration effects would be the only discrepancy between MM and MF cases proved to be incorrect, as differences in force history persisted at \( t/T = 0.25 \) when accelerations were zero. Interestingly, force histories were found to converge near \( t/T = 0.5 \) when accelerations were once again large. This leads us to the second major finding where it was shown that the distribution of trailing-edge vorticity in the wake had a profound impact on force history. The effective incidence \( \alpha_{eff} \) and effective velocity \( |\vec{u}_{eff}| \) at the trailing edge of MF cases lagged the MM cases, due to the time required for the gust to convect along the chord. This resulted in significant variations of trailing-edge vorticity in the wake. The variations in the distribution of wake vorticity coincided with the aforementioned variations in the force histories. This is a significant result, as it demonstrates
that the instantaneous trailing-edge velocity \( \overline{u}_{\text{eff}} \) has a large influence on the contribution of wake effects to force history. This result agrees with the previous study on trailing-edge manipulation performed by Prangemeier et al. (2010).

Acknowledgments

The authors acknowledge the funding provided by the Natural Science and Engineering Research Council of Canada. The authors would like to further acknowledge Mohamed Arif for his technical assistance in the numerical simulations, as well as James Buchholz for his helpful advice.

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