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Growth and separation of a start-up vortex from a two-dimensional shear layer

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The evolution of an isolated line vortex generated by a starting two-dimensional jet is studied experimentally using time-resolved particle image velocimetry. The vortex growth in this current configuration is not linked to any externally imposed length scales or interactions with other vortical structures or walls that could potentially influence vortex growth. A model for the early-stage vortex growth, based on the transport of circulation from the shear layer into the vortex, is proposed and found to agree well with experimental data. The model provides a scaling scheme for vortex growth using shear-layer characteristic velocity and shear-layer thickness. The vortex growth is limited through a gradual separation of the vortex from the feeding shear layer, arising from decreased shear-layer curvature. This phenomenon is linked to a competition between the shear-layer tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer. Finally, a dimensionless number representing this competition is introduced, which in turn is able to describe the gradual separation of the vortex from the shear layer. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4758793]

I. INTRODUCTION

The impulsive start of a shear layer typically initiates roll-up leading to the formation of a vortex. The vortex will grow in size and strength as long as there is a circulation flux into the vortex. Reaching higher levels of vortex circulation on lifting surfaces can be advantageous for lift and thrust production.1, 2 For instance, Milano and Gharib3 showed that for a two-degree-of-freedom flapping plate, maximum average lift was generated for flapping kinematics, which produced leading-edge vortices of maximum circulation. In another example, a cylinder-piston apparatus was used by Krueger4 to conclude that by maximizing the size of the vortex ring, the efficiency of momentum transport could be optimized.

In practice vortex growth is limited. Gharib et al.5 studied the vortex-ring formation through the ejection of fluid from a circular orifice and observed a limit to the vortex-ring growth for piston-stroke-to-diameter ratios of approximately $L/D \geq 4$. Further circulation was observed to be rejected, which accumulated in Kelvin-Helmholtz-type vortices formed in a trailing shear layer behind the vortex ring.5, 6 This interruption of the feeding mechanism is termed the “pinch-off” and has been related to the Kelvin-Benjamin variational principle,5 which states that a steady axis-touching vortex ring has maximum energy compared to other vorticity arrangements with the same impulse. Using a slug model to quantify the shear-layer energy, Gharib et al. showed that the pinch-off occurred at a critical dimensionless time, $T^* = \frac{U_p t}{D} \approx 4$, where $\bar{U}_p$ is the average piston velocity. This critical
formation time was termed the formation number. Shusser and Gharib\(^7\) showed that the pinch-off can be explained equivalently in terms of the translational velocity of the vortex ring exceeding the shear-layer velocity due to self induction. Based on this latter explanation, strategies to delay the pinch-off were proposed by temporarily varying the piston exit diameter.\(^8, 9\)

The limitation of vortex growth has been observed also in configurations involving moving objects either from rest or with periodic motion. Jeon and Gharib\(^10\) observed a saturation of the vortex circulation in the wake of a circular cylinder starting from rest. The saturation occurred after the cylinder had moved four cylinder diameters \((d)\), i.e., a formation time of \( Ut/d = 4\), where \(U\) is the average cylinder velocity. A similar behavior was observed for finite aspect ratio plates moving from rest\(^11\) and for a plate rotating around one of the plate edges.\(^12\) In the latter case, formation times on the order of 0.4–0.9 were reported, smaller than observed in previous studies.\(^10, 11\)

In the search for a unifying principle, Dabiri proposed a formation number as \(\hat{T} = C\Gamma_1/UD \approx 4\), where \(C\) is a configuration-based constant, \(\Gamma_1\) is the vortex strength, and \(U\) and \(D\) are the shear-layer feeding velocity and length scale, respectively.\(^1\) However, several recent studies indicate a need for a reassessment of the concept of universal formation number. For the case of a plunging airfoil Rival \textit{et al.}\(^2\) observed a saturation of the leading-edge vortex at a formation number of \(\hat{T} = 4\). However, higher formation numbers, up to \(\hat{T} = 6\), were observed for plunging airfoils in tandem configuration.\(^13\)

The notion of a universal formation number implies that for a configuration of length scale \(D\), the vortex is unable to grow beyond a given limit. The length scale \(D\) is typically geometry-related, e.g., nozzle opening, cylinder diameter, plate width, or airfoil chord. We refer to these length scales associated with the geometry of the vortex generator as “natural” length scales.

In the studies by Pedrizzetti\(^14\) and Domenichini,\(^15\) the vortex formation from a two-dimensional orifice was investigated. Interestingly, a pinch-off process was not observed, but rather the vortex remained attached to the orifice edge by a shear layer whose velocity was larger than the convective velocity of the vortex. The same behavior was observed by Afanasyev for a two-dimensional stratified flow generated by a finite-size planar nozzle.\(^16\)

Although in the studies by Pedrizzetti and Domenichini no comment on the relationship between pinch-off and length scales of any kind is made, their results suggest that the existence of a natural length scale -in this case nozzle size- is not a sufficient condition for pinch-off. However, the question as to whether the existence of a natural length scale is a necessary condition for a limit to vortex growth remains unanswered. Thus, the purpose of the present work is to investigate the formation process of a two-dimensional line vortex in isolation in which there are no length scales externally imposed by the vortex-generation apparatus. An approximation to the two-dimensional vortex generation was simulated through the start up of a planar two-dimensional jet. The jet flow is generated by a dielectric barrier discharge (DBD) plasma actuator.\(^17\) The plasma actuator was convenient due to its ability to provide a rapid shear layer on demand. In contrast to complex, three-dimensional vortex formation associated with flapping flight, e.g., spanwise influence on leading-edge vortex formation as studied by Beem \textit{et al.},\(^18\) the current study focuses on a two-dimensional vortex-formation process with no wall (wing) or vortex interaction (tip/trailing edge).

A model is proposed for the early-stage vortex formation. The model is based on Kaden’s\(^19\) vortex growth formulation and takes into account non-steady flow variations. This growth model will be shown to be consistent with experimental data obtained with particle image velocimetry (PIV). In addition to investigating the growth process of the vortex, the separation mechanism of the vortex from the shear layer is also discussed.

II. EXPERIMENTAL SETUP AND METHODOLOGY

A. Vortex generation

The two-dimensional planar jet is generated using a dielectric barrier discharge plasma actuator. The actuator consists of two electrodes separated by a dielectric material taped on the surface of a base plate. When a high alternating voltage is applied between the electrodes, a charge build-up on the dielectric surface occurs and causes ionization of the surrounding fluid molecules. The charged
molecules are accelerated in the electromagnetic field and by collision with neutral molecules they transfer momentum into the fluid. In quiescent air, this leads to a starting jet flow with a maximum velocity above the wall surface rolling up into a vortex (Fig. 1), as also observed by Kotsonis et al.\textsuperscript{20} By adjusting the high voltage of the plasma actuator, it is therefore possible to adjust the maximum velocity of the shear layer.

Figure 1 shows a schematic of the plasma actuator. The electrodes were installed very close to the trailing edge of a flat plate in order to avoid wall influence on the vortex generation. The trailing edge of the plate was sharpened to 10 degrees to fix flow separation at the plate trailing edge. The base plate material was black acrylic glass to eliminate laser-light reflections. The plasma actuator spanned the entire 0.15 m plate width. The electrode aspect ratio was chosen as 30:1 to ensure two-dimensionality. Further details regarding the electrical specifications of the plasma actuator are described in Kriegseis et al.\textsuperscript{17} The flat plate with plasma actuator was mounted on a three-axis manual traverse and placed in a 1.1 m $\times$ 0.5 m $\times$ 0.5 m sealed container made out of transparent acrylic glass, as shown in Fig. 2(a).

FIG. 1. Layout of plasma actuator for the impulsive generation of an isolated two-dimensional line vortex. Note wall-jet and free-jet profiles shown for sake of clarification.

FIG. 2. (a) Experimental setup showing the laser light plane, two high-speed cameras, and the plasma actuator mounted on a manual traverse unit placed inside acrylic glass container. The flow is from left to right. (b) Schematic of the two fields of view. The larger field of view (80 mm $\times$ 50 mm) was used to characterize the vortex growth while the smaller field of view (20 mm $\times$ 12 mm) was used to resolve the high velocity gradients inside the shear layer.
TABLE I. Plasma actuator voltage and shear-layer characteristics for different test cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Output voltage, kV</th>
<th>Shear-layer velocity, m/s</th>
<th>Shear-layer thickness, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12.27</td>
<td>3.1</td>
<td>0.96</td>
</tr>
<tr>
<td>B</td>
<td>13.41</td>
<td>3.5</td>
<td>0.99</td>
</tr>
<tr>
<td>C</td>
<td>14.65</td>
<td>4.2</td>
<td>1.06</td>
</tr>
<tr>
<td>D</td>
<td>16.24</td>
<td>5.0</td>
<td>1.10</td>
</tr>
</tbody>
</table>

As summarized in Table I, the shear-layer maximum velocity was controlled by the output voltage and ranged from 3.1 m/s to 5.0 m/s. The operating frequency of the plasma actuator was kept constant at 11 kHz.

B. Particle image velocimetry

Time-resolved PIV was used to study the evolution process of the vortex. Of special interest was the characterization of the vortex growth and shear layer. Since shear-layer thickness was typically one order of magnitude smaller than the vortex core, two cameras were used simultaneously on opposite sides of container: one camera with a larger field of view (FOV = 80 mm × 50 mm) was employed for vortex growth characterization; a second camera with a smaller field of view (FOV = 20 mm × 12 mm) was focused on the shear layer to resolve its velocity profile directly. The camera arrangement and relative locations of the fields of view are shown in Figs. 2(a) and 2(b), respectively. The camera FOVs were aligned with the x-y axes.

A Dantec Dynamics A/S time-resolved PIV system was used for velocity field measurements. A Nd:YLF Litron dual-cavity laser (LDY303) with a maximum power of 70 W and a maximum output energy of approximately 10 mJ per pulse was operated at 3 kHz providing a 527 nm laser light sheet with a thickness of approximately 2 mm. Images were captured by two Phantom V12.1 CMOS cameras at 3 kHz in the double-frame mode with \( dt = 50 \mu s \). To allow for larger particle displacement every first image of the image pairs was used to calculate velocities in the larger field of view. This effectively provided \( dt = 333 \mu s \) (1/3000 s) for the zoomed-out field of view. The cameras had a 1280 × 800 pixel resolution (full sensor used). Micro Nikkor 105 mm lenses were used for both cameras, albeit at different distances from the laser sheet. Image correlations were carried out using DynamicStudio v3.14 software. The vector fields in both fields of view were calculated using an adaptive correlation (with two refinement steps starting from 128 × 128 pixel) and a final 32 × 32 pixel interrogation window size with 50% overlap. A 3 × 3 filter was used to lightly smooth the vector fields in order to more clearly define the vortex core.

The accuracy of measured velocity field was estimated to lie below 1% of the maximum recorded velocity, assuming a maximum sub-pixel interpolation accuracy of 0.2 pixel, see Raffel et al.\textsuperscript{21} Subsequently, random errors in the vorticity and circulation could be estimated to be \( \Delta \omega_{\text{max}}/\Gamma = \pm 0.01 \) and \( \Delta \Gamma/\Gamma_{\text{max}} = \pm 0.025 \), respectively, where \( \Gamma \) is the average vortex core radius, \( u_{\text{max}} \) is the maximum shear-layer velocity, and \( \Gamma \) is the vortex circulation. A central differencing scheme with second-order accuracy was used to estimate the velocity gradients. This led to a systematic error in circulation and shear-layer thickness which is estimated to be less than \( \pm 5\% \) in both cases. To ensure repeatability, the experiments were repeated four times for one representative case. The overall uncertainty in circulation (95% confidence level) was lower than \( \pm 4\% \) among these repeated tests.

A Dantec Dynamics A/S high-volume liquid seeding generator (model 10F03) was used to generate seeding particles at approximately 1 \( \mu m \) in diameter from the DEHS (Di-Ethyl-Hexyl-Sebacat) solvent. The particle image size was approximately 4 pixels in the zoomed-in field of view and \( \sim 1 \) pixel for the zoomed-out field of view. An even distribution of particle sub-pixel displacement around zero was checked so as to ensure the data were not affected by peak locking.
The PIV system and plasma actuator were triggered to start simultaneously using a digital signal output from LabVIEW through a NI USB-6210 data acquisition system.

C. Measurement of shear-layer thickness and velocity

The shear-layer thickness, $D$, was characterized by the vorticity thickness given by Brown and Roshko\textsuperscript{22} as

$$D(t) = \frac{u_{\text{max}}(t) - u_o(t)}{(\partial u/\partial y)_{\text{max}}},$$

where $u_{\text{max}}$ is the maximum velocity in the shear layer and $u_o$ is the velocity at the outside edge of the shear layer. The shear-layer maximum velocity and thickness were obtained from the velocity profile at the streamwise location of the vortex center at each instant in time (along the dashed line in Fig. 1). Since in the present experimental arrangement the shear-layer velocity and thickness do not vary significantly in time, representative quantifications of $D(t)$ and $u_{\text{max}}$ are given by the time-averaged value, as presented in Table I together with their standard deviations for different test cases (statistics obtained from over 30 realizations).

D. Measurement of vortex circulation and vortex size

The vortex circulation was calculated using the surface integral of vorticity normal to the measurement plane as

$$\Gamma = \sum_{i,j} \omega_{i,j} \Delta A_{i,j},$$

where $\omega_{i,j}$ and $\Delta A_{i,j}$ are the vorticity and quarter-area of each PIV interrogation area (note the 50% overlap in PIV correlations). While, for instance, the $\lambda_2$ criterion is an objective vortex identification scheme, it had the effect of excluding parts of the vortex resulting in underestimation of the total vortex circulation. To obtain an accurate estimate of vortex size and strength, a threshold-based algorithm was used instead. The vorticity threshold was set at 0.1 of the peak vorticity measured at each operating plasma actuator voltage. This threshold proved suitable to exclude the shear-layer vorticity in the calculation of vortex-core strength. Since the plasma actuator remained on during a given test, some of the vorticity in the flow field was associated with the shear layer. To exclude these vorticity “patches” detached from the vortex core, the threshold criterion was used to accept only the simply connected vortex patch about the center of rotation in the calculation of vortex-core strength. The conclusions remained unaffected when the calculations were repeated using other vorticity thresholds in the range $0.1 \pm 0.03$ of the peak vorticity. The vortex size was obtained by calculating the area of the simply connected vorticity region. The radius of the vortex was then taken as the radius of a circle with an area equal to that of the vortex.

III. EARLY-STAGE VORTEX DEVELOPMENT

Figure 3 shows a smoke visualization sequence of vortex evolution in time for Case D. In the current experiments, the plasma actuator continued to operate during the entire measurement.
sequence. The start-up phase of the plasma actuator is one order of magnitude faster (less than 3 ms) than the time scale associated with the growth of the vortex (~20 ms to reach maximum growth). Due to the presence of the plate in the start-up phase of the jet, initially the shear layer rolls up only on one side forming one vortex growing in size and circulation. As shown in the last image of the visualization sequence of Fig. 3, a second weaker vortex with clockwise rotation is generated further downstream of the plate trailing edge. This vortex forms well after the evolution phase of the counter-clockwise vortex of interest and therefore is not further studied.

Figure 4 shows a typical $u$-component velocity distribution through the center of the vortex along the line A-A' at time $t = 22$ ms and for case C. For subsequent analysis, the center of the vortex was chosen as the point of maximum vorticity inside the vortex core. Velocity profiles are shown for each of the two fields of view in Fig. 4. Note that when using the larger field of view, the velocities in the high-velocity gradient zone of the shear layer are underestimated due to the lower spatial resolution.

The jet consists of two parallel shear layers with opposite signs of vorticity. The vortex is fed by the circulation flux from the upper shear layer with positive (counter-clockwise) vorticity. On the upper side the velocity distribution exhibits four regions marked in Fig. 4. In the first region (zone 1) the velocity decreases from $u_{\text{max}}$ to a value $u_o$. The velocity stays relatively constant at $u_o$ in the second zone between the shear layer and the core of the vortex, resulting in a zone of low vorticity (zone 2). As presented by the statistics in Table II, it was experimentally observed that $u_o/u_{\text{max}} = 0.5 \pm 0.04$. The third zone is the core of the vortex (zone 3) followed by a zone in which a transition towards the outer quiescent fluid takes place (zone 4).
### TABLE II. Statistics of $\frac{u_c}{u_{\text{max}}}$ for different test cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.54</td>
<td>0.04</td>
</tr>
<tr>
<td>B</td>
<td>0.53</td>
<td>0.04</td>
</tr>
<tr>
<td>C</td>
<td>0.49</td>
<td>0.05</td>
</tr>
<tr>
<td>D</td>
<td>0.49</td>
<td>0.05</td>
</tr>
</tbody>
</table>

On the lower side of the jet the vorticity is clockwise as the velocity rapidly decreases from $u_{\text{max}}$ to zero (zone 5). This velocity gradient is due to the no-slip boundary condition upstream of the plate trailing edge and due to quiescent outer flow downstream of the trailing edge. The two opposite-signed shear layers (zones 1 and 5) are evident in the sample vorticity contour of Fig. 4(b) obtained from the zoomed-in field of view. The absolute vorticity level is larger within the lower (clockwise) shear layer as the velocity gradient occurs within a rather narrow distance. The flow structure after the passage of the vortex is akin to typical wall jets consisting of two layers of opposite-signed vorticity. Accurate characterization of the mean flow field is presented in Moreau.23

As shown in the visualization sequence in Fig. 3, as the vortex evolves, it convects downstream in the horizontal direction as well as away from the shear layer in the vertical direction. Figure 5 shows the $x$-location of the center of the vortex as a function of time for different test cases. Note that the stepwise behavior is due to the limited PIV spatial resolution for the large field of view. The average convective velocity of the vortex in the streamwise direction ($u_c$) was estimated by a linear regression through the recorded positions of the vortex center as a function of time (dashed lines in Fig. 5). As indicated in Fig. 5, for each test case, this velocity was found to scale with the maximum shear-layer velocity as $u_c = (0.12 \pm 0.01)u_{\text{max}}$. The convection behavior of the vortex can be explained through a potential flow model by considering induced velocities of the two opposite-signed vortex sheets of the shear layer. According to the Biot-Savart relation, the counter-clockwise vorticity of the shear layer tends to induce a velocity on the vortex in the negative $x$-direction. However, the lower half of the shear layer (clockwise rotation) induces a velocity on the vortex in the positive $x$-direction. Since the vorticity is higher on the lower half of the shear layer the net induced velocity is positive. To examine this argument, the convective velocity of the vortex due to the presence of the two parallel shear layers with a vorticity distribution similar to the present experiment was calculated using the Biot-Savart relation,

$$u_c = \frac{1}{4\pi} \int \frac{\vec{\omega} \times \vec{\zeta}}{||\vec{\zeta}||^3} dV,$$

(3)

FIG. 5. The $x$-position of the vortex core as a function of time. The slope of the solid lines represents the mean convective speed of the vortex ($u_c$) obtained using the Biot-Savart relation.
where $\vec{\omega}$ is the vorticity vector, $\vec{\zeta}$ is the vector pointing from the vortex-sheet segment towards the vortex center as depicted in the upper left corner of Fig. 5, and $dV$ is the differential element of volume. The convective velocities obtained from Biot-Savart are represented by the slopes of the solid lines in Fig. 5. The origin of the lines is chosen arbitrarily (without loss of generality). At earlier stages of vortex formation, the convective speed of the vortex is slightly over-estimated by Biot-Savart relation, but later the predicted speed is very close to experimental data.

### A. Model for early-stage vortex development

Two main effects contribute to the vortex growth: transport of vorticity from the shear layer via mass flux into the vortex, as suggested by Kaden,° as well as viscous diffusion. Following Kaden,° i.e., assuming the diffusion to be negligible, the size of the vortex may be estimated by the time integral of mass flow rate into the vortex (Fig. 6). Given that the flow is two-dimensional and incompressible, the volume flow rate per unit width, $\dot{Q}$, can be related to the vortex radius $r$, where

$$\int_0^t \dot{Q}(t) dt = \pi r^2(t). \tag{4}$$

Considering the convective velocity of the vortex, $u_c$, the volume flow rate per unit width entering the vortex from the counter-clockwise vorticity shear layer may be written in terms of a velocity distribution:

$$\dot{Q}(t) = \int_0^D \left( u(t, y) - u_c(t) \right) dy. \tag{5}$$

Equations (4) and (5) are time-dependent and are presented in general form. In the present case, however, the shear-layer velocity distribution and thickness vary negligibly in time, allowing Eqs. (4) and (5) to be re-written in terms of time-averaged values. Considering that the mean shear-layer velocity can be approximated by the average of the maximum shear-layer velocity, $u_{max}$, and with the velocity outside the shear layer, $u_o$, Eqs. (4) and (5) become

$$\bar{\dot{Q}} \approx \pi r^2(t), \tag{6}$$

$$\bar{\dot{Q}} \approx \left[ \bar{u}(t, y) - \bar{u}_c(t) \right] \bar{D}(t) \approx \left( \bar{u}_{max} + \bar{u}_o \right) \bar{D} - \bar{u}_c. \tag{7}$$

Furthermore, based on the experimental observations, it is possible to scale $u_o$ and $u_c$ with $u_{max}$:

$$\bar{u}_o \approx 0.5 \bar{u}_{max}, \tag{8}$$

$$\bar{u}_c \approx 0.12 \bar{u}_{max}. \tag{9}$$

Finally, by substituting Eqs. (7)–(9) into Eq. (6), an expression for the growth of the vortex radius in time is obtained

$$r(t) = \sqrt{\frac{0.63 \bar{u}_{max} \bar{D}}{\pi} t}. \tag{10}$$
Equation (10) suggests that the vortex grows with the square root of time and that the vortex size is a function of thickness and velocity of the shear layer. Having obtained the radius, the vortex strength is determined as the line integral of the velocity around the core of the vortex, i.e., \( \Gamma = \int \vec{u} \cdot d\vec{l} \). By approximating the vortex area to be of circular shape and assuming a uniform-magnitude velocity distribution around the vortex core in the frame of reference moving with the vortex, the line integral may be written in a more simplified form:

\[
\Gamma(t) \approx 2\pi r(t) u_t(t),
\]

where \( u_t \) is the rotational velocity at the edge of vortex core in the frame of reference moving with the vortex. Since a uniform velocity distribution around the vortex core is assumed, it is sufficient to have \( u_t \) at only one point around the vortex core. In the present setup, the velocity in the region between the vortex core and the shear layer (zone 2 in Fig. 4), \( u_o \), can be taken as the velocity at the edge of the vortex core. However, as depicted in Fig. 4, to transform this velocity from the stationary frame of reference into the frame of reference of the vortex, \( u_t \) should be considered as

\[
u_t = u_o - u_c.
\]

After substitution of \( r(t) \) from Eq. (10) into Eq. (11), and applying Eqs. (8), (9), and (12) an expression for the vortex circulation in time as a function of shear-layer characteristics is obtained

\[
\Gamma(t) \approx 2\pi r(t)(\overline{u_o} - \overline{u_c}) \approx 0.76\pi u_{max} \sqrt{0.63\frac{u_{max}D}{\pi t}},
\]

which is a linear relationship between dimensionless circulation, \( \tilde{\Gamma} \), and dimensionless time \( \tilde{t} \), independent of geometric length scales of the problem. This scaling approach is in line with Dabiri’s formulation of the formation time using shear-layer characteristics.

**B. Scaling of vortex growth**

Considering the relationship for circulation from Eq. (13), it is possible to obtain a scaling scheme for vortex growth based on shear-layer characteristics by scaling both sides of Eq. (13) by \( u_{max}D \) to obtain

\[
\tilde{\Gamma} = \frac{\Gamma(t)}{u_{max}D} \approx 1.07 \sqrt{\frac{\overline{u_{max}t}}{D}} = 1.07\tilde{t},
\]

**IV. RESULTS**

Figure 7 illustrates the estimated vortex radius as a function of time for the test cases summarized in Table 1. The growth predictions from the proposed model (Eq. (10)) are also shown and agree with experiments in the early vortex growth stage. The diffusion rate, \( \sqrt{4\nu t} \), typical of, for instance, a Lamb-Oseen vortex with constant circulation, has been plotted for comparison, suggesting that the growth rate associated with viscous diffusion is much slower compared to the influx from the shear layer. The strong agreement between the model and the experimentally measured vortex radius also indicates that inviscid entrainment does not significantly contribute to the growth of the vortex.

Generally, the vortex growth is faster when the maximum velocity in the shear layer is increased. However, the vortex size is observed to asymptotically reach a plateau, thus deviating from the model. Due to this asymptotic behavior it is not possible to specify a precise time associated with vortex growth limit. However, the approximate onset of the plateau in vortex size for each case has been marked by a dashed line in Fig. 7.

A very similar behavior is observed for the growth of circulation (Fig. 8). The start of the plateau in the vortex strength coincides with that for the vortex size. This plateau indicates that the flux of circulation into the vortex decreases through a gradual process.
FIG. 7. Vortex radius for different test cases together with predictions by the proposed growth model (solid black lines). The growth rate due to viscous diffusion, $\sqrt{4\nu t}$ (Lamb-Oseen model), is included for comparison. The dashed line marks the approximate onset of the vortex-growth plateau.

A. Separation of vortex from shear layer

The limit in vortex growth shown in Figs. 7 and 8 indicates that the circulation flux from the shear layer into the vortex is eventually terminated. It is hypothesized that this limit is related to the change in the shear-layer curvature, or “flattening” of the shear layer away from the vortex in time, leading to a gradual separation of the vortex from the shear layer. This phenomenon is shown schematically in Fig. 9(a). Flow patterns and vorticity contours at two time steps corresponding to early-stage vortex formation (left) and a time after the vortex separation from the shear layer (right) are presented in Figs. 9(b) and 9(c), respectively. The increased shear-layer radius of curvature ($\kappa$) is evident from the change in the streamline patterns. Prior to the separation, the streamlines are directed towards the core of the vortex, whereas after separation, some of the streamlines are directed away from the core of the vortex.

The “flattening” of the shear layer is due to the competition between the tendency of fluid particles inside the shear layer to maintain an inertial trajectory in the streamwise direction and the induced velocity (upwards) by the vortex. The induced velocity by the vortex on the shear layer is a function of vortex strength, $\Gamma$, and is also inversely related to the distance, $S$, between the core of the vortex and the shear layer. The shear-layer maximum velocity, $u_{\text{max}}$, can be considered as the characteristic parameter associated with the shear layer’s tendency to remain in the streamwise direction. This competition can be therefore expressed in terms of a dimensionless number,

$$\Gamma^* = \frac{\Gamma}{u_{\text{max}} S}. \quad (15)$$

FIG. 8. Vortex strength for different test cases together with predictions by the proposed growth model (solid lines). The dashed line marks the approximate onset of the vortex-growth plateau and is the same as in Fig. 7.
As long as the induction from the vortex is sufficiently large compared to $u_{\text{max}}$, i.e., large $\Gamma^*$, the shear-layer curvature will remain close to that of the vortex core radius. At the same time the circulation-containing fluid from the shear layer will also be drawn towards the vortex core. In the current configuration, however, the vortex core travels away from the shear layer with a faster rate than the circulation growth. This process leads to a gradual decrease in the contribution of vortex-induced velocity on the shear layer, i.e., a decrease in $\Gamma^*$, hence a “flattening” of the shear layer and a separation of the vortex from the feeding shear layer. Figure 10 presents $\Gamma^*$ as a function of time for the four test cases showing the decrease of $\Gamma^*$ in time. The separation occurs at approximately $\Gamma^* \approx 1.5$ for all four test cases.

Based on the above argument the following relationship describing the connection between the radius of curvature of the shear layer ($\kappa$), vortex radius ($r$), and dimensionless number $\Gamma^*$ may be proposed based on scaling principles:

$$\frac{r}{\kappa} \propto \frac{\Gamma}{u_{\text{max}}S}.$$  \hspace{1cm} (16)

In the current experimental arrangement, the distance $S$ grows faster than the vortex size, $r$, and continues to increase when the vortex size has already reached a plateau. This is evident from the plot of $r/S$ as a function of time in Fig. 11 and indicates that $S$ depends on the global flow field rather than directly on vortex growth itself. The dependence of spacing $S$ on the flow field provides the possibility of delaying the separation of the vortex from the shear layer by manipulating the distance $S$ through a change in the global flow conditions. The significance of distance $S$ on vortex growth has also been observed in the numerical study by Mohseni et al., where a delay in vortex ring...
pinch-off was achieved by gradually increasing the orifice diameter so that the vortex ring would grow away from the symmetry axis. In other configurations involving vortex growth, such as the case of a plunging airfoil, the free-stream velocity may have an impact on the variations in distance $S$ with a favorable/unfavorable influence on vortex growth.\textsuperscript{2} Alternatively, in the present setup the separation may potentially be delayed by decreasing the shear-layer velocity in time so that the induced velocity from the vortex remains competitive with the shear-layer velocity for a longer time period.

Since the radius of curvature is difficult to determine directly from the measurements, an approximate measure of shear-layer “flattening” can be obtained by considering the geometric parameter $S/H$, where $H$ is the distance between the center of the vortex and the shear layer in the horizontal direction, as depicted in the upper-right corner of Fig. 12. The $S/H$ ratio for test cases A-D is presented in Fig. 12, where for clarity cases B-D have been shifted vertically by $S/H = 1$, 2, and 3, respectively. As expected, a drop in $S/H$ is observed, which coincides with the plateau in vortex size and circulation (Figs. 7 and 8). The approximate time of the drop in $S/H$ is marked by the dashed line in Fig. 12.
FIG. 12. Parameter $S/H$ provides a measure of the shear-layer curvature. The drop in the shear-layer curvature corresponding to the separation of the shear layer from the vortex is marked by the dashed line (the same line as in Figs. 7 and 8). The drop occurs at the same approximate time of the plateau in vortex growth. For clarity, cases B-D have been plotted with a vertical offset of $S/H = 1, 2, \text{and } 3$, respectively.

V. CONCLUDING REMARKS

The evolution of an isolated line vortex generated by a starting two-dimensional jet was studied experimentally using time-resolved particle image velocimetry. Based on Kaden’s\textsuperscript{19} vortex-growth formulation, a model was proposed for the early-stage vortex formation and was shown to be consistent with the experimental results. Through this model, a scaling scheme for the growth stage of the vortex using the shear-layer characteristic velocity and the shear-layer thickness was obtained. The vortex growth was, however, observed to be limited. The limitation was found to be linked to a competition between the shear-layer tendency to remain in the streamwise direction and the induced velocity from the vortex on the shear layer itself. It was proposed that this competition is a function of the vortex strength, $\Gamma$, spacing between the shear layer and the vortex, $S$, and the shear-layer characteristic velocity, $u_{\text{max}}$, and therefore can be described using a dimensionless number, $\Gamma^{*} = \Gamma / u_{\text{max}} S$. As the contribution of the velocity induced by the vortex relative to the shear-layer characteristic velocity is reduced, the shear layer “flattens” and consequently separates from the vortex through a gradual process. It is observed that separation occurs at $\Gamma^{*} \approx 1.5$. We hypothesize that the separation process may be hindered by maintaining the vortex-induced velocity comparable with the shear-layer characteristic velocity through (i) controlling the spacing $S$ by external means such as a co-flow or (ii) a temporal decrease in the shear-layer velocity. Pedrizzetti’s\textsuperscript{14} simulation of a two-dimensional vortex dipole may in fact be a confirmation of this hypothesis. Unlike the present study, in Pedrizzetti\textsuperscript{14} the induction from the opposite-signed mirror vortex (through Biot-Savart) causes the distance between the shear layer and the vortex to remain relatively constant. Data extracted from Pedrizzetti\textsuperscript{14} for the low Reynolds number case suggest that the non-dimensional circulation remains larger than $\Gamma^{*} = 10$. Despite the uncertainty in data extraction, these approximations of $\Gamma^{*}$ are much higher than in the present study at the time of plateau ($\approx 1.5$). This might explain the continuation of attachment of the vortex to the shear layer in the numerical studies of Pedrizzetti and Domenichini as well.

In addition to $\Gamma^{*}$, turbulence is also expected to play a role in the separation process. Unlike previous studies of Pedrizzetti and Afanasyev, the flow in the present study appears to be turbulent ($\text{Re} > 1500$ based on shear-layer velocity and mean vortex diameter). In turbulent flows instabilities are more easily amplified. At sufficiently low $\Gamma^{*}$ the connection of the vortex to the shear layer is susceptible to perturbations and ready to break off. Hence, these perturbations can accelerate the disconnection of the vortex from the shear layer. A more detailed analysis of the influence of turbulence on vortex evolution is subject to future studies.
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