A conditional analysis of spanwise vortices within the lower atmospheric log layer

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ARTICLE INFO

Article history:
Received 16 November 2012
Received in revised form
22 March 2013
Accepted 3 May 2013
Available online 20 June 2013

Keywords:
Atmospheric turbulence
Spanwise vortices
Coherent structures
Conditional analysis

ABSTRACT

The current study is an initial step towards the full characterization of coherent structures within the turbulent atmospheric boundary layer (ABL). A conditional-analysis technique to identify spanwise vortices is applied on data acquired throughout January–August 2012. This study begins with an approximation of the ABL stability and thickness (δ). Subsequently the measured Reynolds stresses are compared to those measured by Klebanoff (1955) over a flat plate and similarity formulations developed by Marusic and Kunkel (2003). The ABL data is shown to be neutral with a thickness of approximately 1000 m. The statistical descriptors of the turbulent fluctuations are found to qualitatively agree with Klebanoff (1955) and Marusic and Kunkel (2003). From the conditional analysis, it is found that the height of spanwise vortices follows a Rayleigh-like distribution with a mean and standard deviation of 0.065δ and 0.038δ, respectively. The circulation distribution of spanwise vortices is found to be bimodal about Γ/Upδ = 0 with a standard deviation of 0.036Upδ. The magnitude of the positive peak of the bimodal distribution tends to be 80–100% that of the negative peak, which demonstrates that prograde vortices are more frequent than retrograde vortices.

1. Introduction

This work is a first step in characterizing coherent structures within the lower atmospheric log layer, which is key towards further understanding turbulent fluctuations and coherent structures within this region. A more coherent understanding of turbulence within the lower log layer of the atmosphere is valuable for several applications such as wind-turbine control, managing gusts during the operation of small autonomous aircraft, such as micro aerial vehicles, and wind loadings on civil structures. The quantification of coherent structures within the log region of the ABL would also characterize the influence of the Reynolds number over several orders of magnitude. Coherent structures within canonical turbulent boundary layers (TBLs) have been typically quantified through the measurement of planar vortices such as spanwise vortices. The study of spanwise vortices has led to an improved understanding of turbulence mechanics and to improved computational models (Robinson, 1989). Thus, the current study looks to quantify the strength and orientation of ABL spanwise vortices to achieve similar ends.

Earlier research into atmospheric turbulence has generally focused on the time-averaged characteristics of the velocity fluctuations. For example, as demonstrated in Kaimal (1973), the power spectra of the velocity fluctuations and their cross-correlations follow normalized curves that are functions of the Richardson number. Similar relations involving the fluctuation spectra have been determined and are well documented in various texts such as Lumley and Panofsky (1964) and Kaimal and Finnigan (1994). A major drawback of time-averaging is that it is an indiscriminate filter such that quantification of the major energy-containing coherent structures within the wind field is not possible. The need to obtain a better temporal and spatial characterization of the flow has prompted a shift towards alternate measurement techniques to quantify the entire flow field. In meteorological studies, radar and lidar measurements have provided insightful data. For example, Lin et al. (2008) used lidar measurements, and Kropfi and Kohn (1978) used radar measurements, to characterize the nature of spanwise roller vortices within the ABL. However, meteorological studies such as these focus primarily on coherent structures that are on the order of the boundary layer thickness (δ), while the current study focuses on structures of a much smaller scale.

In contrast, using measurement techniques such as particle imaging velocimetry (PIV), studies on canonical TBLs have investigated turbulent coherent structures on orders much smaller than δ. One structure widely found within canonical TBLs is the hairpin vortex. The hairpin vortex is a ubiquitous coherent structure that explains several aspects of canonical TBL flow, and has been
observed both experimentally and computationally at various Reynolds numbers; see Robinson (1989, 1991) for a review of the early research performed on hairpin vortices. Hairpin vortices were first observed three-dimensionally by Head and Bandyopadhyay (1981). However, recent studies such as Christensen and Adrian (2001) have explained how early, qualitative, two-dimensional observations of turbulent coherent motions (Nychas et al., 1973 and Praturi and Brodkey, 1978) were in fact of hairpin vortices Hairpin vortices have been observed throughout the entire TBL and scale proportionally with the height of the head from the shearing flat surface; see Zhou et al. (1999), Adrian et al. (2000) and Adrian (2007).

It can be shown that the turbulent kinetic-energy budget of neutral ABLs is equal to that of canonical TBLs; see Lumley and Panofsky (1964). Furthermore, a similarity formulation developed in Marusic and Kunkel (2003) developed for canonical TBLs of high Reynolds number has been shown to accurately describe the Reynolds stresses of ABL flows; see Kunkel and Marusic (2006). It is expected that the Reynolds stresses measured in the current study will also agree with the similarity formulation from Marusic and Kunkel (2003).

The shared traits between ABLs and canonical TBLs would also give credence to our expectation of similar coherent structures within ABLs and canonical TBLs. One must, however, be careful in assuming that the characteristics of coherent structures within ABLs are comparable to characteristics of structures found within canonical TBLs. Due to the size of the atmosphere, entire coherent structures within the ABL have yet to be fully quantified. Studies that have visualized coherent structures within ABLs have had to compromise between qualitatively measuring full-scale structures, as done in Hommem and Adrian (2003) via smoke visualizations of full-scale hairpin packets, and quantitative measurements within a small (1 m × 0.5 m) field-of-view with PIV (Morris et al., 2007). Another issue is that studies focused on coherent structures within canonical TBLs have been performed at low Reynolds numbers ($R_{\infty} = 10^4$–$10^5$) whereas ABL flow exhibits Reynolds numbers several orders greater ($R_{\infty} = 10^6$–$10^8$). Thus it would be incorrect to draw connections from lab experiments as it remains unclear how the properties of turbulent flow scale with Reynolds number; see Degraaff et al. (1999) and Smits et al. (2011) for examples of Reynolds number scaling issues.

Since velocimetry techniques are still maturing for the measurement of time- and spatially-resolved velocity fields in ABLs, one must make do with wind masts to quantify coherent structures within ABLs in a meaningful way. Wind masts at best provide complete, high temporal resolution velocity measurements at only a limited number of data-acquisition points along their height. Scarabino et al. (2007) attempted to characterize hairpin vortices within a neutral ABL using data collected from a wind mast by Sterling et al. (2006). Using a conditional analysis (CA) technique, velocity fluctuations of extreme gust events were classified into three groups and were then ensemble averaged. One of the ensemble groups exhibited vortex-like flow, which led Scarabino et al. (2007) to fit a hairpin-vortex model onto the ensemble average. In this regard the study had limited success: although the hairpin-vortex model accounted for sharp changes in streamwise velocity, it failed to satisfactorily capture the simultaneous vertical fluctuations.

It is believed that the shortcomings of the hairpin-vortex model in Scarabino et al. (2007) were due to some oversimplifications. For example, it was assumed that the gust event coincided with the impingement of the center of the hairpin-vortex head on the mast. The study did not consider placing the impingement prior to or after the gust event. Furthermore, the assumption that the 6 s-long ensemble averages were comprised of a single hairpin vortex seems too basic. The ensemble averages observed by Scarabino et al. (2007) could have been induced by a series of hairpin vortices, which would induce a complicated flow field.

The wide range of structures within TBLs and the complex flow fields they induce compels the argument that coherent structures must be studied first in terms of their strength and orientation prior to investigating how they contribute to gust events and Reynolds stresses. Thus the current study takes an alternate approach to that in Scarabino et al. (2007). Rather than identifying gust events within the data and attempting to fit a certain coherent structure to their ensemble average, events that meet specific criteria consistent with a certain coherent structure are identified instead. Velocity fields are then determined for each specific event and are compared to experimental data. In this way, the orientation and strength of coherent structures within the lower log region of the ABL can be characterized, statistically analyzed and compared to those within canonical TBLs.

In the current study, high-speed velocity measurements from two ultrasonic sensors located on a wind mast are analyzed to characterize atmospheric spanwise vortices. Previous studies have shown that the occurrence-frequency distribution of spanwise vortex-center height is independent of Reynolds number. However, modest increases in Reynolds number have resulted in an increased ratio of retrograde to prograde vortices; see Wu and Christensen (2006). Since ABLs have characteristically high Reynolds numbers, it is expected that the occurrence-frequency distribution of spanwise vortex-center height will be comparable to lab studies, while the ratio of retrograde to prograde vortices will increase significantly in comparison to low Reynolds number studies.

Since spanwise vortices are but components of larger vortical superstructures, the superstructure itself can interfere with the detection of the spanwise vortex. Thus, a heuristic model drawn from canonical TBLs is presented in Section 2 to demonstrate that ABL spanwise vortices are detectable from the mast data in spite of the superstructure’s presence. Afterwards, a CA technique is then developed to identify spanwise vortices using the two available ultrasonic sensors.

In Section 3 the measured Reynolds stresses are compared to measurements taken within a canonical TBL and subsequently to a similarity formulation from Marusic and Kunkel (2003). Finally, the results of the CA are presented. The occurrence-frequency distributions of normalized circulation ($\Gamma/|U_\infty|$) and normalized vortex-center height ($a/\delta$) of spanwise-vortex events are discussed, and the model’s performance in identifying vortices is then evaluated.

2. Materials and methods

This section describes the wind site as well as the sensors fitted onto the wind mast. This is then followed by a heuristic model illustrating that spanwise-vortex events are identifiable despite the signal interference caused by the encompassing vortical superstructures.

2.1. Experimental setup

Eight data sets were acquired from January 2012 to August 2012 from a 50 m wind mast erected on university land. A schematic of the wind mast, with the approximate positioning of its sensors, is provided in Fig. 1(a). A photograph depicting the mast and the surrounding terrain is shown in Fig. 1(b). The mast is instrumented with five cup anemometers and wind vanes, as well as a two-component ultrasonic anemometer, which are all used to quantify the boundary-layer profile and mean wind direction. Spanwise vortices are characterized using the measurements
taken by the two three-component ultrasonic anemometers (3CUS). The 3CUS sensors are located at 40 m and 50 m heights. To determine the ABL stratification for each data set, the Obukhov length $\Gamma$ was calculated using data recorded by a meteorological measurement station located 200 m west of the wind mast (Hayashi et al., 2010).

The mast is indicated by a square. The mast’s guide wires are indicated by thick black lines and are approximately 30 m in length. The area contained between line segments AB and CD indicates a small climb, which ascends from southwest to northeast. Point L indicates the lowest point in the gully which is 5 m lower. Point T indicates the position of the nearest tree to the mast. The images are taken from Google©Maps.

2.2. Spanwise-vortex signal interference

Two-dimensional spanwise vortices are not stand-alone structures. Rather, spanwise vortices are two-dimensional fluctuation signatures belonging to larger vortical superstructures. It is possible that the superstructure can obscure the signal generated by its spanwise-vortex component. The following heuristic model demonstrates that the spanwise-vortex components of such superstructures can be detected independently from wind-mast data.

It is hypothesized here that similar to canonical TBLs, hairpin-vortices remain the most pervasive vortical superstructures within ABLs. Based on this hypothesis, a simplified hairpin-vortex model is developed below. Consider an idealized hairpin vortex being convected towards the mast as shown in Fig. 3. It is approximated as a potential vortex composed of three line segments: two inclined vortex lines represent the legs, while a vortex line running along the spanwise direction represents the hairpin-vortex head. To simulate the proximity of the ground, a mirror-image hairpin vortex is placed below the ground plane. The dimensions of the hairpin vortex are defined by the parameters $W$, $H$, and $L$, which represent the leg spacing, the head height, and base length of the hairpin vortex, respectively. The head has a circulation of $\Gamma_h$, while the legs have a circulation of $\Gamma_l$. The hairpin vortex is convected towards the wind mast at a convective velocity of $U_0$. One of the 3CUS sensors is located at a height $H_s$ off the ground and the base of the hairpin vortex, respectively. The head has a circulation of $\Gamma_h$, while the legs have a circulation of $\Gamma_l$. The hairpin vortex is convected towards the wind mast at a convective velocity of $U_0$. One of the 3CUS sensors is located at a height $H_s$ off the ground and the base of the hairpin vortex, respectively. The head has a circulation of $\Gamma_h$, while the legs have a circulation of $\Gamma_l$. The hairpin vortex is convected towards the wind mast at a convective velocity of $U_0$. One of the 3CUS sensors is located at a height $H_s$ off the ground and the base of the hairpin vortex, respectively. The head has a circulation of $\Gamma_h$, while the legs have a circulation of $\Gamma_l$. The hairpin vortex is convected towards the wind mast at a convective velocity of $U_0$. One of the 3CUS sensors is located at a height $H_s$ off the ground and the base of the hairpin vortex, respectively. The head has a circulation of $\Gamma_h$, while the legs have a circulation of $\Gamma_l$. The hairpin vortex is convected towards the wind mast at a convective velocity of $U_0$. One of the 3CUS sensors is located at a height $H_s$ off the ground and the base of the hairpin vortex, respectively. The head has a circulation of $\Gamma_h$, while the legs have a circulation of $\Gamma_l$. The hairpin vortex is convected towards the wind mast at a convective velocity of $U_0$. One of the 3CUS sensors is located at a height $H_s$ off the ground and the base of the hairpin vortex, respectively.

To evaluate the velocity $\vec{V}$ induced by the hairpin-vortex at the sensor, the Biot–Savart law is employed

$$d\vec{V} = \frac{\Gamma}{4\pi} \frac{d\vec{I} \times \vec{r}}{\|\vec{r}\|^3},$$

(1)
where \( \overrightarrow{d} \) is an infinitesimal line segment along the hairpin vortex, \( \overrightarrow{T}_i \) is the displacement vector from the sensor to the line segment, and \( \Gamma \) is the local circulation at the line segment. Each vortex line within the hairpin vortex has unique expressions for \( \overrightarrow{d} \), \( \overrightarrow{T}_i \) and \( \Gamma \), as well as specific upper and lower bounds of integration (\( \alpha \) and \( \beta \), respectively) as determined by the directionality of the circulation. The fluctuation velocity measured at the sensor is the sum of contributions of each line segment

\[
\overrightarrow{V}_i = \frac{\Gamma_h}{4\pi} \int_{\alpha}^{\beta} \frac{d\overrightarrow{l}_{12} \times \overrightarrow{r}_{12}}{||r_{12}||^3} + \frac{\Gamma_l}{4\pi} \int_{\alpha}^{\beta} \frac{d\overrightarrow{l}_{34} \times \overrightarrow{r}_{34}}{||r_{34}||^3} + \frac{\Gamma_h}{4\pi} \int_{\alpha}^{\beta} \frac{d\overrightarrow{l}_{45} \times \overrightarrow{r}_{45}}{||r_{45}||^3} + \frac{\Gamma_l}{4\pi} \int_{\alpha}^{\beta} \frac{d\overrightarrow{l}_{56} \times \overrightarrow{r}_{56}}{||r_{56}||^3} + \frac{\Gamma_h}{4\pi} \int_{\alpha}^{\beta} \frac{d\overrightarrow{l}_{61} \times \overrightarrow{r}_{61}}{||r_{61}||^3} + \frac{\Gamma_l}{4\pi} \int_{\alpha}^{\beta} \frac{d\overrightarrow{l}_{63} \times \overrightarrow{r}_{63}}{||r_{63}||^3}.
\]

In Eq. (2) the first and fourth terms represent the velocity induced by the hairpin-vortex head, while the remaining terms represent the velocity induced by the hairpin-vortex legs. Only the integrals that simulate the head of the hairpin vortex can be solved analytically. To avoid this limitation, several simplifications are made to the model based on observations made by studies performed at lower Reynolds numbers. Fig. 4 summarizes these observations on the head height, leg spacing, viscous-core radius and leg-incline angle of hairpin vortices. From the studies summarized in Fig. 4(d) the incline angle of the hairpin-vortex legs is taken as 45°, such that \( H=L \). Although Fig. 4(b) suggests that the spacing of hairpin-vortex legs is around 0.1h to 0.2h, Fig. 4(a) demonstrates that the heads of hairpin vortices have been observed throughout the entire boundary layer. It is implausible that such narrow hairpin structures with legs where the vortical axes are not parallel with the mean flow could extend throughout the entire boundary layer without being significantly distorted. To put the implication of this assumption into perspective, in an ABL with a thickness of \( \delta = 1000 \text{ m} \), 100 m-wide hairpin vortices should also extend 1000 m upwards. Thus, the current study assumes that a hairpin vortex’s width must grow proportionally with the head’s height, which has been observed computationally by Zhou et al. (1999) and is described in Adrian et al. (2000) and Adrian (2007). This assumption implies that \( \Gamma = H \). Finally, Acclar and Smith (1987) have shown that circulation of the legs is approximately one-fifth that of the head with an uncertainty of 40%. Thus, the ratio can range from 0.12 to 0.28, with 0.28 representing a worst-case scenario since the legs are more comparable in strength to the head. The current study assumes that \( \Gamma_H/\Gamma_h = 1/3 \) to avoid underestimating the leg strength.

It is here argued that a statistically significant number of hairpin vortices will impinge upon the wind mast such that the idealized vortex structure can be bisected into roughly two halves. It can be demonstrated that during such events, the spanwise-vortex-like motion generated by the head alone can be considered a signature of a hairpin vortex. Consider a hairpin vortex where \( R_a = -H/2 \) so that the vortex is evenly divided by the mast, and where the head is now impinging on the mast so that \( t = \frac{L}{2} - U_0 t \). A value of 45 m is assigned to \( H \) so that the following analysis is applicable to both 3CUS sensors. At this instant, by taking the magnitude of the spanwise-plane velocity induced by the hairpin-vortex legs and dividing it by the magnitude of the spanwise-plane velocity induced by the hairpin-vortex head, a ratio of leg-induced spanwise-plane velocity \( \overrightarrow{V}_l \) to head-induced spanwise-plane velocity \( \overrightarrow{V}_h \) is determined

\[
\frac{||\overrightarrow{V}_l||}{||\overrightarrow{V}_h||} = \sqrt{\left(V_{23x} + V_{34x} + V_{56x} + V_{61x}\right)^2 + \left(V_{23y} + V_{34y} + V_{56y} + V_{61y}\right)^2}
\]

(3)

Using the aforementioned simplifications made to the geometry of a hairpin vortex, Eq. (3) can be simplified into a function of \( H \) only, as plotted in Fig. 5. The ratio approaches the limit of 0.45 as the head height approaches infinity. Below a height of 100 m, the spanwise-plane velocity induced by the hairpin vortex’s legs is only 40% of that induced by the head. Thus, the current study argues that spanwise-vortex signatures of hairpin vortices can be independently identified from the superstructure at heights below 100 m. Note that the hairpin-vortex model considers only spanwise vortices with negative circulation or prograde vortices, while spanwise vortices with positive circulation (retrograde vortices) are not. A similar model cannot be constructed for retrograde vortices, as the origin of retrograde vortices is not well understood. However, various studies have shown a connection between hairpin vortices and retrograde vortices; see Section 3.2. Thus, the current study assumes that vortical superstructures similar to a hairpin vortex must encompass retrograde vortices and therefore, retrograde vortices are also independently identifiable below 100 m.

2.3. Spanwise-vortex impingement condition

A condition suggestive of spanwise-vortex impingement will now be presented. Spanwise vortices are represented as an infinitely long Vatistas vortex of viscous-core radius \( R \) being convected towards the mast, as shown in Fig. 6. The circulation of the vortex is \( \Gamma \) and the vortex center sits at an arbitrary height \( y=a \). The vortex is convected by the mean local velocity \( U_0 \) at the vortex-center height. To account for ground effect, two counter-rotating vortices are positioned at heights \( +a \) and \( -d \). Fig. 6 also shows the mast with the heights of the top and bottom ultrasonic
sensors labeled as $h_T$ and $h_B$, respectively. If Taylor’s frozen-field hypothesis is assumed, then the vortex’s shape should not change as it convects. This allows $b$ to be set equal to $-U_a(t-t_R)$, where $t_R$ is the instant the vortex center has passed the mast.

The fluctuation field generated by this vortex is given by $u$ and $v$, which are the longitudinal and vertical fluctuations induced by the vortex, respectively. At the top and bottom 3CUS sensors, the vortex induces fluctuations of

$$
\vec{V}(t, h_T) = u(t, h_T) \hat{i} + v(t, h_T) \hat{j},
$$

$$
\vec{V}(t, h_B) = u(t, h_B) \hat{i} + v(t, h_B) \hat{j}.
$$

If the current vortex is far from other vortices, then the fluctuation field is given by

$$
u \left( t, y \right) = \frac{r}{2 \pi} \left( \frac{y-a}{r_n(y, t)^2} - \frac{y+a}{r_p(y, t)^2} \right),
$$

$$
u \left( t, y \right) = \frac{r}{2 \pi} \left( \frac{U_a(t-t_R)}{r_n(y, t)^2} - \frac{U_a(t-t_R)}{r_p(y, t)^2} \right),
$$

where $r_n(y, t)^2$ and $r_p(y, t)^2$ are given by

$$
r_n(y, t)^2 = \sqrt{(y-a)^2 + U_a^2(t-t_R)^2} + R^4,
$$

$$
r_p(y, t)^2 = \sqrt{(y+a)^2 + U_a^2(t-t_R)^2} + R^4.
$$

Fig. 4. (a) Normalized height of hairpin-vortex heads $H/\delta$. (b) Normalized space between hairpin-vortex legs $W/\delta$. (c) Normalized viscous-core radius of hairpin-vortex heads. (d) Incline angle of hairpin-vortex legs from the wall $\arctan(H/L)$ as observed by various studies. The height of the rectangles indicates the value range observed while the width of the rectangles indicates the momentum-thickness Reynolds number range for each respective study. Lighter shaded rectangles indicate that the study observed hairpin vortices while darker shaded rectangles indicate that the study did not make reference to hairpin vortices. The studies which did not refer to hairpin vortices directly are included since the observations and experimental setups are very comparable to those that claim to have observed hairpin vortices.

Fig. 5. The ratio of spanwise-plane velocity magnitudes $|\vec{V}_i|/|\vec{V}_o|$ induced by a hairpin vortex’s legs and head at a 45 m-high sensor as a function of the hairpin vortex’s head height $H$. 

The vortex-induced fluctuations at the sensors can be determined by substituting \( y \) with \( h_T \) and \( h_B \) in Eqs. (6) and (7). By taking frequency-component ratios and setting \( t = t_R \), Eqs. (10) and (11) are obtained

\[
u_{(t_R, h_T)} = \frac{a-h_T}{a-h_B} + \frac{a+h_T}{a+h_B} = f_1(a, R),
\]

\[
u_{(t_R, h_B)} = \frac{1}{\sqrt{(h_T-a)^4 + R^4}} - \frac{1}{\sqrt{(h_T+a)^4 + R^4}} = f_2(a, R),
\]

where \( f_1 \) and \( f_2 \) are only dependent on the heights of the two 3CUS sensors, the vortex-center height and the viscous-core radius, respectively. A value of 0.015 is selected for \( R \), which is in agreement with Fig. 4(c).

Eqs. (10) and (11) define a condition for identifying spanwise vortices. If a spanwise vortex impinges on the mast at time \( t_R \), then the streamwise- and vertical-fluctuation ratios as defined in Eqs. (10) and (11) can only be satisfied for a unique vortex-center height \( a \). That is to say, for instances where the value of \( a \) determined via Eq. (10) is the same value of \( a \) determined via Eq. (11), a spanwise vortex has been identified. \( R \) can then be determined by knowing \( a \) thereby defining Eqs. (6) and (7) completely. Finally the occurrence-frequency distributions of \( a \) and \( R \) can be compared to those observed in canonical TBLs to both validate the model and to gain insight into the differences between the coherent structures observed within ABLs and canonical TBLs.

To prohibit the criterion from identifying anomalous events, Eqs. (6) and (7) evaluated at 40 m and 50 m can be compared to the measured fluctuation time traces in the vicinity of time \( t_R \) so as to reduce the ambiguity of the model. An automated marking scheme was used to test how closely Eqs. (6) and (7) describe the measured time traces. The scheme considered numerical discrepancies between the measured/estimated fluctuations and measured/estimated fluctuation gradients. Each event was given a numeric grade based on the quality of its fit. Finally, for each data set, only the identified events with grades that fell within the upper two quartiles were then considered.

By comparing Eqs. (6) and (7) to the measured time traces, the majority of anomalous events was discounted. However, extended periods of positive or negative streamwise fluctuations with no corresponding vertical fluctuations were occasionally identified as spanwise vortices with large circulations located high above the sensors. It is uncertain if such events are caused by spanwise vortices. An additional criterion could be added to remove the identification of such events. However, since the model has been restricted to only the first 100 m from the ground, such events do not affect the results of the current study.

Figs. 7 and 8 are examples of typical spanwise-vortex events identified using the above criterion. The blue curves represent measured time-traces, while the red curves represent the fluctuations predicted by Eqs. (6) and (7). \( R \) is nearly the same magnitude in both examples (236.2 m²/s and −220.0 m²/s). In Fig. 7, the 50 m and 40 m sensors fall within the viscous core of the spanwise vortex, whereas in Fig. 8, the viscous core only impinges on the 50 m sensor. The function of the viscous core is to keep the spanwise-vortex fit from overshooting the time traces. Without the inclusion of the viscous core, the spanwise-vortex fit would greatly overshoot all measured time trace in Fig. 7, while overshooting the 50 m time traces in Fig. 8 as well.

### 3. Results

This section describes the ABL data set by evaluating important descriptors such as the stability, the thickness and the Reynolds number. Afterwards, the results of the CA are evaluated.

#### 3.1. Description of the ABL data sets

Table 1 summarizes various flow parameters of all eight data sets. The momentum-thickness Reynolds number \( Re \) for all eight sets was much greater than what is typically achieved in laboratory studies. \( Re \) ranged from a minimum of \( 3.2 \times 10^{7} \) to a maximum of \( 21.1 \times 10^{7} \) for the current study while coherent structures within TBLs are typically examined at \( Re \sim 10^{3} \) (see Fig. 4).

The stability of the ABL may be determined by analyzing the mean velocity profile measured by the cup anemometers. In a neutral ABL, the mean velocity profile fits well onto the curve given by Kaimal and Finnigan (1994)

\[
U(y) = \frac{u_s}{\kappa} \ln \frac{y}{y_0}
\]

where \( y \) is the vertical distance from the ground, \( u_s \) is the friction velocity, \( \kappa \) is the von Kármán constant (approximately 0.4) and \( y_0 \) is the friction length. Both \( u_s \) and \( y_0 \) are determined from the curve fit. Table 1 provides \( u_s \) and \( y_0 \) for each data set. The
Fig. 7. A spanwise-vortex event from the January 5th data set. \( a \) and \( \Gamma \) of the vortex are 44.4 m and 236.2 m\(^2\)/s, respectively. Both sensors fall within the viscous core of the vortex. (For interpretation of the references to color in this figure the reader is referred to the web version of this article.)

Fig. 8. A spanwise-vortex event from the January 5th data set. \( a \) and \( \Gamma \) of the vortex are 54.6 m and \(-220.0\) m\(^2\)/s, respectively. Only the top sensor falls within the viscous core of the vortex. (For interpretation of the references to color in this figure the reader is referred to the web version of this article.)
coefficients of determination $R^2$ for each data set are also provided, which are a measure of the quality of the fit. All the $R^2$ values for the eight data sets exceed 0.97, indicating that Eq. (12) is a good descriptor of the mean velocity. The fact that the wind-velocity profiles are well described by a logarithmic fit indicates that the mast sits within the log region (surface layer) of the ABL. Furthermore, if the accurate representation of the mean velocity profile via Eq. (12) can be considered a sufficient criterion for assuming neutral stratification, then the ABL is presumably neutral for all eight data sets. However, the Obukhov lengths $\Lambda$ listed in Table 1 indicate that the majority of the data sets is somewhat unstable, with two sets being highly unstable. Thus, the aforementioned criterion for assuming neutral stratification is not reliable.

The majority of the sets exhibits unstable stratification ($\Lambda < 0$), which is likely caused by the data sets being collected in the afternoon. However, given that the majority of the data sets exhibits $|\Lambda| \approx 200$ m, it is expected that the stratification for all eight data sets is still near-neutral.

The ABL thickness $\delta$ for each data set was determined using the approximation from Blackadar and Tennekes (1968)

$$\delta = \frac{0.2 u_\infty}{2 \Omega \sin \phi}$$

Table 1

<table>
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<th>Date</th>
<th>Time</th>
<th>Symbol</th>
<th>$u_\infty$ (m/s)</th>
<th>$y_0$ (cm)</th>
<th>$R^2$</th>
<th>$\delta$ (m)</th>
<th>$U_0$ (m/s)</th>
<th>$\frac{R_{2/10}}{10^7}$</th>
<th>$\Lambda$ (m)</th>
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<td>21.3</td>
<td>21.1</td>
<td>-581</td>
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<tr>
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<td>68.7</td>
<td>0.982</td>
<td>1199</td>
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<td>11.8</td>
<td>-20</td>
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<td>0.913</td>
<td>1103</td>
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<td>10.3</td>
<td>216</td>
</tr>
<tr>
<td>06/27</td>
<td>15:20–17:20</td>
<td>green</td>
<td>0.56</td>
<td>1.0</td>
<td>0.999</td>
<td>988</td>
<td>16.2</td>
<td>9.1</td>
<td>-252</td>
</tr>
<tr>
<td>07/03</td>
<td>15:40–16:20</td>
<td>cyan</td>
<td>0.43</td>
<td>0.5</td>
<td>0.957</td>
<td>766</td>
<td>13.0</td>
<td>5.5</td>
<td>-209</td>
</tr>
<tr>
<td>07/18</td>
<td>13:30–15:30</td>
<td>blue</td>
<td>0.38</td>
<td>1.2</td>
<td>0.996</td>
<td>678</td>
<td>10.5</td>
<td>4.2</td>
<td>-207</td>
</tr>
<tr>
<td>07/23</td>
<td>15:20–17:30</td>
<td>purple</td>
<td>0.56</td>
<td>1.2</td>
<td>0.999</td>
<td>980</td>
<td>15.7</td>
<td>8.9</td>
<td>-601</td>
</tr>
<tr>
<td>08/05</td>
<td>10:10–12:40</td>
<td>green</td>
<td>0.33</td>
<td>0.4</td>
<td>0.978</td>
<td>582</td>
<td>5.7</td>
<td>3.2</td>
<td>-30</td>
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</tbody>
</table>

Fig. 9. Mean flow velocities and mean wind directions of all eight data sets. In (a)-(c) the markers indicate the mean velocities recorded by the cup anemometers during the measurement periods. The error bars ascribed to each marker indicate the standard deviation of the recorded velocity. Certain markers are without error bars because the standard deviation is less than the marker width. In (d)-(f) the vertical lines indicate the mean wind direction recorded by the wind vanes during the measurement periods. The red curves/histograms in (a)/(d), (b)/(e), and (c)/(f) show the conditional nine-month trend during intervals where the 50 m cup anemometer measured 6–8 m/s, 10–12 m/s and 12–14 m/s, respectively. The red curves in (a)-(c) present the nine-month trend in the form of Eq. (12). $u_\infty$ and $y_0$ of each nine-month trend are provided in the bottom-right corner. For clarity, the markers in (a)-(c) have been shifted from the actual heights of the respective sensors, which are indicated by the black horizontal lines. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)
Eq. (13) provides only a rough approximation for $\delta$. Since the mean value of $\delta$ is 977 m, $\delta$ is taken nominally as 1000 m for all data sets. The freestream velocities $U_0$ listed in Table 1 were approximated by evaluating Eq. (12) at 1000 m. Since the 3CUS sensors are spaced 10 m apart, the viscous-core radius employed by the conditional model was taken as 12 m instead of 10 m to avoid possible singularities within the model.

Fig. 9 shows the mean velocities and wind directions measured by the cup anemometers and wind vanes, respectively. The eight data sets have been classified into three groups based on the mean velocity measured by the 50 m cup anemometer. Fig. 9(a) and (d), Fig. 9(b) and (e) and Fig. 9(c) and (f) present the mean velocity and wind direction for data sets where the 50 m cup anemometer measured an average velocity of 6–8 m/s, 10–12 m/s and 12–14 m/s, respectively. A conditional nine-month trend based on the same criteria is plotted on all figures for comparison. These three groups are hereafter referred to as low-, mid- and high-wind sets. The mean-velocity measurements of the low-wind sets tend to differ from the nine-month trend by more than a single standard deviation, while both the mean wind direction of the low-wind data sets as well as the nine-month trend exhibit a great deal of scatter. This suggests that a higher level of uncertainty is associated with the low-wind sets. In contrast, the majority of the mean-velocity measurements for mid- and high-wind sets falls within a standard deviation of the nine-month trend. Furthermore the mean-wind direction for the mid- and high-wind sets all align with the mean-wind direction of the nine-month trend, west-south-west, where the peak is now noticeably more pronounced. Thus at the test site, the higher wind speed measurements taken from wind mast tend to have a higher level of certainty.

Fig. 10 shows various statistical descriptors of the turbulent fluctuations as measured by the 3CUS sensors (observed flow), as well as those measured in a canonical TBL over a flat plate by Klebanoff (1955). The discrepancy between the observed flow and TBL flow may be due to the difference in Reynolds number. The TBL-flow data was collected at a Reynolds number $Re_{\theta} = 4.9 \times 10^4$ while the Reynolds number for this ABL data is on the order of $Re_{\theta} \sim 10^5$. At heights of $0.05 \delta$ the statistical descriptors of turbulent fluctuations at different Reynolds numbers do not collapse onto a single curve when normalized by outer parameters such as $\delta$ and $U_0$; see Degraaff et al. (1999). The discrepancy could have also been caused by an underestimation of $\delta$. By increasing $\delta$, $U_0$ must correspondingly increase, which would move the observed results in Fig. 10 towards the bottom-left of their respective graphs.

Fig. 11 plots the Reynolds stresses measured by the 3CUS sensors for all eight data sets. The Reynolds stresses have been normalized by $u_\tau$ and are plotted against wall-unit height $y^+ = y/(u_\tau / \nu)$, where $\nu$ is the kinematic viscosity of air. Furthermore, the values predicted by Marusic and Kunkel (2003) are indicated by the black lines in Fig. 11(a), (b) and (d). The effect that stratification has on the Reynolds stresses is made clear in Fig. 11. The Reynolds stresses as measured by the current study appear in two bands: a band with Reynolds stresses that agree closely with Marusic and Kunkel (2003) and exhibit values of $-\overline{uu'}/(u_\tau)^2$ that are near unity; and a band with Reynolds stresses that do not agree with Marusic and Kunkel (2003) and exhibit values of $-\overline{uu'}/(u_\tau)^2$ slightly above unity. Sets with neutral or stable stratification agree with Marusic and Kunkel (2003) and exhibit $-\overline{uu'}/(u_\tau)^2 - 1$. In contrast, sets with unstable stratification exhibit

![Fig. 10](image-url)  
*Fig. 10. Normalized root-mean-square values of turbulent fluctuations in the (a) streamwise, (b) vertical and (c) spanwise directions, as well as the normalized correlation between streamwise and vertical fluctuations (d). The black curves are taken from measurements over a flat plate performed by Klebanoff (1955) while the markers indicate values observed by the current study. The markers correspond to the data sets as described in Table 1. For clarity the markers are staggered vertically. The horizontal lines indicate the heights of the two 3CUS sensors, while the upper and lower shaded regions indicate if a measurement was taken by the upper or lower sensor, respectively.*
Reynolds stresses greater than what is predicted by Marusic and Kunkel (2003) and exhibit $\overline{w'w''}/u_*^2$ slightly larger than 1. It is to be expected that the Reynolds stresses would strengthen as the stratification becomes increasingly unstable. It should be noted, however, that the 05/29 set is highly unstable yet falls within the neutrally/stably stratified band. Furthermore, the absolute value of $\Lambda$ for the 07/23 set suggests that the ABL was neutrally stratified during data collection. However, the 07/23 set falls within the unstably stratified band.

### 3.2. Results of the conditional analysis

The CA was performed on all eight data sets. Table 2 provides the number of events identified for each data set, as well as the mean number of events identified per minute of each data set. Roughly 1000 events were identified per data set, or, approximately 10 events-per-minute. The number of events identified by the model is sufficient enough to be statistically significant, and the events are also sufficiently spaced apart in time to ensure that consecutive events are not interfering.

Fig. 12 shows the occurrence-frequency distributions of vortex-center height $a$ and circulation $\Gamma$ for all eight data sets. $a$ and $\Gamma$ have been normalized by $\delta$ and $U_0\delta$, respectively. Since the conditional model is considered accurate up until a height of 100 m, the distributions of $a$ have been normalized so that the area from $0\delta$ to $0.1\delta$ is unity. The distributions of $\Gamma$ have been normalized so that the area is unity underneath the entire curve. The distribution of $a$ resembles a Rayleigh probability distribution. The two peaks located at 0.04$\delta$ and 0.05$\delta$ are likely caused by the positioning of the two 3CUS sensors, whose heights are indicated by two vertical lines in Fig. 12(a). The mean of $a$ is 0.06$\delta$, 0.07$\delta$, and 0.07$\delta$ for low-, mid- and high-wind sets, respectively. The standard deviation of $a$ is 0.03$\delta$, 0.04$\delta$, and 0.03$\delta$ for low-, mid- and high-wind sets, respectively. In contrast, $\Gamma$ exhibits a bimodal distribution about $U_0\delta = 0$ regardless of wind speed, while its standard deviation was found to be 0.08$U_0\delta$, 0.07$U_0\delta$, and 0.03$U_0\delta$ for low-, mid- and high-wind sets, respectively.

Various studies performed on canonical TBLs have found similar distributions for $a$. From a numerical simulation on a canonical boundary layer ($Re = 670$) by Spalart (1986), Robinson (1989) found that spanwise vortices also follow a Rayleigh distribution with an average height of approximately 0.2$\delta$. From experiments run at $Re_\theta = 1120$, Smith and Lu (1990) also found that the vortex-center heights follow a Rayleigh distribution with an average height of 0.06$\delta$. Finally, from the experiments of Wu and Christensen (2006), it was found that over a large Reynolds number range ($4000 \leq Re_\theta \leq 11000$) the center height of the spanwise vortices followed a Rayleigh distribution with a mean of approximately 0.06$\delta$. The results from Smith and Lu (1990) and Wu and Christensen (2006) have been plotted in Fig. 12(a) for comparison. It is noted that below 0.02$\delta$ and beyond 0.1$\delta$, the distributions in $a$ determined by the current study underestimate the occurrence frequency when compared to the aforementioned distributions. One possible cause for this is that the signals generated by distant spanwise vortices are influenced by inherent system noise. As the centers of the spanwise vortices move away from the sensors, the vortex circulation must be large such that
they can be detected. Otherwise, the amplitude of the vortex-induced signal is comparable to the amplitude of the system’s noise, thereby becoming impossible to detect.

The two peaks that occur near $0.04\delta$ and $0.05\delta$ can be directly attributed to the selection of a constant viscous-core radius. The January 5th data was processed using a viscous-core radius of 6 m, 12 m and 24 m. An increase in $R$ results in the peaks moving outwards, away from the sensors. The selection of a constant viscous-core radius also has adverse effects on the criterion’s ability to identify near-ground vortices. The doubling of $R$ resulted in a halving of the number of identified vortices within the first 20 m above the ground.

The bimodal distribution of $\Gamma$ as observed by the current study is more prominent in the mid- and high-wind data sets, yet for low-wind data sets the peaks appear very close together or not at all. The lack of wind during the low-wind data sets may have caused the bimodal distribution to be subdued. Low winds would correspond to weaker signals induced by the spanwise vortices, which could have been obscured by noise.

The bimodal distribution of $\Gamma$ has also been previously observed. Negative circulations are associated with prograde spanwise vortices that induce negative streamwise fluctuations below the vortex center while positive circulations are associated with retrograde spanwise vortices that cause positive fluctuations below the vortex center. Wu and Christensen (2006) found that the spanwise swirling strength, in which the reciprocal represents the period required for a fluid particle to rotate about its local spanwise axis, followed a bimodal distribution about $0$ ($Re_\theta = 8830$). Robinson (1989) measured the circulation of spanwise vortices directly and found that the distribution was bimodal, yet also found that prograde vortices are more abundant than retrograde vortices. A higher population in prograde vortices translates into a bimodal distribution where the negative peak is taller than the positive peak. The results from Robinson (1989) have been plotted in Fig. 12(b)–(d) for comparison.

**Table 2**

<table>
<thead>
<tr>
<th>Data set</th>
<th>01/05</th>
<th>05/29</th>
<th>06/26</th>
<th>06/27</th>
<th>07/03</th>
<th>07/18</th>
<th>07/23</th>
<th>08/09</th>
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<tr>
<td>Number of events</td>
<td>791</td>
<td>1238</td>
<td>1279</td>
<td>695</td>
<td>439</td>
<td>1430</td>
<td>752</td>
<td>1831</td>
</tr>
<tr>
<td>Events-per-minute</td>
<td>6.6</td>
<td>10.2</td>
<td>14.4</td>
<td>11.4</td>
<td>10.8</td>
<td>12.0</td>
<td>6.0</td>
<td>12.2</td>
</tr>
</tbody>
</table>

**Fig. 12.** Occurrence-frequency distributions of (a) vortex-center height $a$ for all eight data sets and (b)–(d) circulation $\Gamma$ for low-, mid- and high-wind data sets, respectively. $\Gamma$ is normalized by $U_0\delta$, while $a$ is normalized by $\delta$. $\delta$ is assumed to equal 1000 m, while $U_0$ is determined by evaluating equation (12) at 1000 m. The two vertical lines in (a) indicate the positions of the 3CUS sensors. Distributions of vortex-center height and circulation as observed within canonical TBLs have been provided for comparison.
from Wu and Christensen (2006) agree with Robinson (1989), which demonstrate that prograde vortices are more common near the shearing surface, while retrograde vortices are more common higher up in the boundary layer. From the current study, the majority of data sets that exhibited a bimodal distribution also exhibited a negative peak that was taller than its positive counterpart. However, the current study observed positive peaks with heights ranging between 80% and 100% of the height of the negative peak. In one of the mid-wind distributions the positive peak actually overtook the negative peak. Thus in the current study the fraction of retrograde vortices was observed to be much greater than observed by Robinson (1989) or Wu and Christensen (2006). This is likely due to the large increase in Reynolds number. Note that Wu and Christensen (2006) observed that the fraction of retrograde vortices increases with Reynolds number.

Generally, hairpin vortices can only explain the presence of prograde vortices, while the origin of retrograde vortices remains poorly understood. However, some speculations have linked retrograde vortices to hairpin vortices as well. One such possibility is the pinching off and reconnecting of hairpin-vortex legs, which form localized ring-like structures as observed by Moin et al. (1986), Melander and Zabuski (1988) and Smith et al. (1991). Another possibility was reported by Yang et al. (2001), who observed the formation of retrograde vortices just below and upstream of strong hairpin vortices. The distortion of hairpin vortex structures from an idealized shape, which occurs at modest Reynolds numbers (see Schroeder et al., 2011), could also be the cause of these retrograde vortices. However, the data collected from the wind-mast cannot quantify entire vortical superstructures like hairpin vortices. Thus, although the results of the current study suggest an increased presence of the above-described mechanisms, this cannot be concluded in the current study and demands more attention.

4. Conclusions

The current study provides an initial step towards the characterization of turbulent coherent structures within the lower atmospheric log layer. A CA identifying spanwise-vortex events was performed on eight data sets collected throughout 2012. Spanwise vortices were modeled as Vatistas vortices with viscous-core radii of 12 m or approximately 0.01δ. The eight data sets examined in the current study were classified as low-, mid- and high-wind data sets based on the mean wind speed measured at 50 m. It was found that the Reynolds stresses of the neutrally and stably stratified ABL data sets agreed with Reynolds-stress values expected from canonical TBLs at much lower Reynolds numbers. In contrast, ABL sets of unstable stratification exhibited Reynolds-stress values larger than those found within the canonical TBLs. The CA found that the occurrence frequency of vortex-center height followed a Rayleigh distribution for all eight data sets, with a mean and standard distribution of 0.065 δ and 0.034δ, respectively. The circulation followed a bimodal distribution about U0δ = 0 with a standard distribution of 0.036U0δ, and with a negative peak that tended to be slightly taller than the positive peak. The peaks of the bimodal distribution were less apparent for the low-wind data sets than for the mid- and high-wind sets. The height distribution determined by the CA agreed with previous studies. Although a bimodal distribution in circulation has been observed in previous studies, the positive peak was observed to be only 25% as tall as the negative peak, whereas the current study observed positive peaks with heights ranging between 80% and 100% of that of the negative peaks. This signifies an increased fraction of retrograde vortices, which has been attributed to the high Reynolds numbers of the data sets. Overall, the model and measurement technique have demonstrated the ability to characterize spanwise-vortex motions within the ABL below heights of 0.16 near the sensor locations, offering promise for future studies on the coherence of gusts in the ABL.

Acknowledgments

The authors wish to thank the generous financial and technical support of Genivar Wind in the development and operation of the wind mast, and Dr. Masaki Hayashi for providing the data collected by his meteorological measurement station. Thanks also go to the Natural Sciences and Engineering Research Council, the Queen Elizabeth II Scholarship and the Canadian School of Energy and Environment for their financial backing.

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